Regular Implementability of nD Behaviors

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Abstract

In this paper, we consider the problem of implementability and regular implementability of a nD behavior by interconnecting a given plant behavior with a suitable controller through a control variable (partial interconnection). In the 1D case, regular implementability by partial interconnection can be characterized in terms of the regular implementability of desired system variable behavior from the original system variable behavior by full interconnection. It turns out that a similar result is not valid in the nD case. However we show that it is possible to obtain an alternative characterization in terms of the regular implementability of a suitably defined control variable behavior.

1 Introduction

As is well known, the notion of system interconnection is the basis for control within the framework of the behavioral approach. In this setting, a general control problem can be stated as follows. Given a plant behavior $\mathcal{B}$ and a control objective corresponding to a desired behavior $\mathcal{B}^d$, find a controller behavior $\mathcal{B}^c$, within a certain controller class, such that the behavior resulting from the interconnection of $\mathcal{B}$ and $\mathcal{B}^c$ coincides with $\mathcal{B}^d$.

An extensive study of control by interconnection of 1D and nD systems is carried out in [3], where apart from the usual interconnections also the concept of *extended interconnection* is introduced. The basic idea of extended interconnection is to introduce additional variables and to construct controllers in the extended variable space. The additional variables can be thought of as being latent variables which are internal to the system and cannot be directly affected by means of (non-extended) interconnection. By allowing restrictions to be placed on such internal variables a larger set of controlled behaviors can in principle be achieved.

An alternative approach introduced in [1], [2], [4] and [5] for 1D systems consists in assuming that the variables are partitioned into to-be-controlled variables w and control variables c. The following control problem is then considered. Given a plant behavior $\mathcal{B}_{(w,c)}$ with partitioned variable (w, c) and a desired controlled w-behavior $\mathcal{B}_w^d$, find a controller behavior $\mathcal{B}_c^c$, which is obtained by restricting only the control variables c, such that its interconnection with the plant yields $\mathcal{B}_w^d$. If this problem is solvable $\mathcal{B}_w^d$ is said to be *implementable from* $\mathcal{B}_{w,c}$ *by interconnection through* c.

The purpose of this paper is to investigate implementability for the case of multidimensional (nD) systems.

2 Notation and preliminaries

Since in the sequel we deal with behaviors in different variables, in order to avoid confusion we will always indicate the behavior variable as a subscript. More concretely, the notation $\mathcal{B}_a$ means that the behavior variable in $\mathcal{B}$ is a. In this case we also say that $\mathcal{B}$ is an a-behavior.

2.1 nD kernel behaviors

We consider nD behaviors $\mathcal{B}_a$ that can be described by a set of linear partial difference or partial differential equations, i.e.,

$$\mathcal{B}_a = \ker E(\sigma_1, \dots, \sigma_n) := \{a \in \mathcal{U} | Ea = 0\},$$

where $\mathcal{U}$ is the trajectory universe, the σ_i's are either the usual nD shifts or the elementary nD partial differential operators and $E(s_1, \dots, s_n)$ is a nD polynomial matrix. We refer to these behaviors as *kernel behaviors*. Since the matrix E uniquely specifies the behavior $\mathcal{B}_a$, we also say that E is a *representation* of $\mathcal{B}_a$ and use the notation $\mathcal{B}_a = \mathcal{B}(E)$.

In case the variable a is partioned as $a = (w, c)$, we will consider the representation matrix E to be partitioned accordingly as $E = [R \quad -M]$. This clearly corresponds to writting the equation $Ea = 0$ as $Rw = Mc$.

Instead of characterizing $\mathcal{B}_a$ by means of a representation matrix E, it is also possible to characterize it by means of its *orthogonal module* $\mathrm{Mod}(\mathcal{B}_a)$, which consists of all the nD polynomial rows r such that $\mathcal{B}_a \subset \mathcal{B}_a(r)$, and can be shown to coincide with the polynomial module generated by the rows of E.

2.2 Behavior interconnection

Given two behaviors $\mathcal{B}_w^1$ and $\mathcal{B}_w^2$ with the same variable, we define their *full interconnection* as the intersection $\mathcal{B}_w^1 \cap \mathcal{B}_w^2$. Moreover, we say that this full interconnection is *regular* if

$$\mathrm{Mod}(\mathcal{B}_w^1) \cap \mathrm{Mod}(\mathcal{B}_w^2) = \{0\}. \tag{2.1}$$

If $\mathcal{B}_w^1 = \mathcal{B}(R^1)$ and $\mathcal{B}_w^2 = \mathcal{B}(R^2)$, then $\mathcal{B}_w^1 \cap \mathcal{B}_w^2 = \mathcal{B}(\begin{bmatrix} R^1 \\ R^2 \end{bmatrix})$ and it constitutes a regular interconnection if and only if

$$\mathrm{rank} \begin{bmatrix} R^1 \\ R^2 \end{bmatrix} = \mathrm{rank}\, R^1 + \mathrm{rank}\, R^2, \tag{2.2}$$

where the ranks are taken as ranks of nD polynomial matrices.

In addition to the (full) interconnection of behaviors with the same variable, we will also consider the (partial) interconnection of a (w, c)-behavior and a c-behavior, [1], [2], [4] and

[5]. Given two behaviors $\mathcal{B}^1_{(w,c)}$ and $\mathcal{B}^2_c$, we define their *interconnection through* c as the (w,c)-behavior given by the intersection $\mathcal{B}^1_{(w,c)} \cap \mathcal{B}^2_{(w=free,c)}$, where

$$\mathcal{B}^2_{(w=free,c)} := \{(w,c) | w \in \mathcal{U}_w \ \wedge \ c \in \mathcal{B}^2_c\}$$

and $\mathcal{U}_w$ is the universe of the w-trajectories. We will say that the interconnection of $\mathcal{B}^1_{(w,c)}$ and $\mathcal{B}^2_c$ through c is *regular* if the full interconnection of $\mathcal{B}^1_{(w,c)}$ and $\mathcal{B}^2_{(w=free,c)}$ is regular.

If $\mathcal{B}^1_{(w,c)} = \mathcal{B}([R \ \ -M])$ and $\mathcal{B}^2_c = \mathcal{B}(C)$, then the interconnection of these two behaviors through c is regular if and only if

$$\operatorname{rank} \begin{bmatrix} R & -M \\ 0 & C \end{bmatrix} = \operatorname{rank} [R \ \ -M] + \operatorname{rank} C, \tag{2.3}$$

where again the ranks are taken as ranks of nD polynomial matrices.

2.3 Control by interconnection

The problem of control by full interconnection can be stated as follows. Given a plant with behavior $\mathcal{B}_w$ and a desired control objective corresponding to a behavior $\mathcal{B}^d_w \subset \mathcal{B}_w$, find a controller with behavior $\mathcal{B}^c_w$, within a certain controller class, such that the behavior $\mathcal{B}_w \cap \mathcal{B}^c_w$ (resulting from the full interconnection of the plant and the controller) coincides with $\mathcal{B}^d_w$. If this control problem is solvable, the (control objective) behavior $\mathcal{B}^d_w$ is said to be *implementable from* the (plant) behavior $\mathcal{B}_w$ *by full interconnection.*

If the interconnection $\mathcal{B}_w \cap \mathcal{B}^c_w$ is regular, then $\mathcal{B}^d_w$ is said to be *regularly implementable from* $\mathcal{B}_w$ *by full interconnection* and the corresponding controller $\mathcal{B}^c_w$ is called a *regular full controller.*

Assume now that the behavior $\mathcal{B}_w$ is given as the w-behavior of a (w,c)-behavior $\mathcal{B}_{(w,c)}$, i.e. $\mathcal{B}_w = \Pi_w(\mathcal{B}_{(w,c)}) := \{w | \exists\, c : (w,c) \in \mathcal{B}_{(w,c)}\}$. Regarding c as a control variable and w as the variable to be controlled, the following control problem can be formulated. Given a control objective corresponding to a behavior $\mathcal{B}^d_w \subset \mathcal{B}_w$, find a controller with behavior $\mathcal{B}^c_c$ such that $\mathcal{B}^d_w$ is the w-behavior of the interconnection of $\mathcal{B}_{(w,c)}$ and $\mathcal{B}^c_c$ through c, i.e., find $\mathcal{B}^c_c$ such that

$$\mathcal{B}^d_w = \Pi_w(\mathcal{B}_{(w,c)} \cap \mathcal{B}^c_{(w=free,c)}).$$

If this control problem is solvable, $\mathcal{B}^d_w$ is said to be *implementable from* $\mathcal{B}_{(w,c)}$ *by interconnection through* c.

If the full interconnection of $\mathcal{B}_{(w,c)}$ and $\mathcal{B}^c_{(w=free,c)}$ is regular, $\mathcal{B}^d_w$ is *regularly implementable from* $\mathcal{B}_{(w,c)}$ *by interconnection through* c, and $\mathcal{B}^c_c$ is called a *regular controller.*

The notion of regular implementability of nD behaviors by full interconnection has been studied in [3] under the name of achievability by regular interconnection. On the other hand, implementability and regular implementability of 1D behaviors by interconnection through a control variable c have been completely characterized in [1]. In the following sections we will study these properties for the nD case.

3 Implementability

In this section we will show that the results obtained in [6] on the implementability of 1D behaviors by interconnection through a control variable c are still valid for the nD case.

Let $\mathcal{B}_{(w,c)}$ be a nD (w,c)-behavior with description

$$Rw = Mc, \tag{3.4}$$

$\mathcal{B}_w := \Pi_w(\mathcal{B}_{(w,c)})$ the corresponding w-behavior and $\mathcal{B}_w^d$ a sub-behavior of $\mathcal{B}_w$. Define the hidden behavior $\mathcal{B}_w^h$ as the w-behavior of $\mathcal{B}_{(w,c)} \cap \{(w,c)|w \in \mathcal{U}_w \wedge c = 0\}$. Clearly,

$$\mathcal{B}_w^h = \ker R. \tag{3.5}$$

Theorem 3.1. *With the previous notation, the following statements are equivalent.*

1. *$\mathcal{B}_w^d$ is implementable from $\mathcal{B}_{(w,c)}$ by interconnection through c.*
2. *$\mathcal{B}_w^h \subset \mathcal{B}_w^d$.*

Proof

$1 \Rightarrow 2$: Assume that 1 holds. This means that there exists a nD polynomial matrix K such that $\mathcal{B}_w^d$ is the w-behavior described by

$$\left\{ \begin{array}{lcl} Rw & = & Mc \\ Kc & = & 0. \end{array} \right. \tag{3.6}$$

Since the trajectories in $\{(w,c)|c = 0 \wedge w \in \mathcal{B}_w^h\}$ satisfy (3.6), it immediately follows that $\mathcal{B}_w^h = \Pi_w(\{(w,c)|c = 0 \wedge w \in \mathcal{B}_w^h\}) \subset \mathcal{B}_w^d$, proving that 2 is verified.

$1 \Rightarrow 2$: Assume now that 2 holds true. We wish to show that there exists a controller behavior $\mathcal{B}_c^c = \ker K$, for some suitable nD polynomial matrix K, such that $\mathcal{B}_w^d$ is the w-behavior described by (3.6).

Note first that the assumption that $\mathcal{B}_w^h \subset \mathcal{B}_w^d$ implies that there exists a nD polynomial matrix F such that $\mathcal{B}_w^d = \ker FR$. We will next show that $K := FM$ yields the desired controller. This amounts to check that $\mathcal{B}_w^d$ is the w-behavior described by

$$\begin{bmatrix} R \\ 0 \end{bmatrix} w = \begin{bmatrix} M \\ FM \end{bmatrix} c. \tag{3.7}$$

But this is obvious, since $[F \quad -I]$ is a minimal left annihilator (MLA) of $\begin{bmatrix} M \\ FM \end{bmatrix}$ and therefore the w-behavior corresponding to (3.7) is $\ker[F \quad -I] \begin{bmatrix} R \\ 0 \end{bmatrix} = \ker FR = \mathcal{B}_w^d$. ■

4 Regular implementability

As concerns regular implementability, it is shown in [1] that the following holds for the 1D case.

Theorem 4.1. *If $\mathcal{B}_w^d$ is implementable from $\mathcal{B}_{(w,c)}$ by interconnection through c, then the following statements are equivalent.*

1. *$\mathcal{B}_w^d$ is regularly implementable from $\mathcal{B}_{(w,c)}$ by interconnection through c.*
2. *$\mathcal{B}_w^d$ is regularly implementable from $\mathcal{B}_w$ by full interconnection.*
3. *$\mathcal{B}_w = (\mathcal{B}_w)^{controllable} + \mathcal{B}_w^d$.*

Here, $(\mathcal{B}_w)^{controllable}$ denotes the controllable part of $\mathcal{B}_w$, i.e., the largest controllable sub-behavior of $\mathcal{B}_w$ (see [7] for a more detailed definition).

The relation between the second and third conditions of this theorem has also been studied for the nD case in [3], where it was shown that $2 \Rightarrow 3$, but $3 \not\Rightarrow 2$. As illustrated in the next example, also the equivalence between 1 and 2 is no longer valid in the nD case as $1 \not\Rightarrow 2$.

Example 4.1. Let $\mathcal{B}_{(w,c)}$ be the 2D (w,c)-behavior described by the equation

$$w = \begin{bmatrix} \sigma_2 - 1 \\ 1 - \sigma_1 \end{bmatrix} c,$$

or equivalently, by

$$\begin{bmatrix} 1 & 0 & 1-\sigma_2 \\ 0 & 1 & \sigma_1 - 1 \end{bmatrix} \begin{bmatrix} w \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and let $\mathcal{B}_w^d$ be the zero behavior. Define the controller $\mathcal{B}_c^c = \ker 1$ (which corresponds to forcing the control variable c to be equal to zero). The interconnection of $\mathcal{B}_{(w,c)}$ and $\mathcal{B}_c^c$ through c, given by the equation

$$\begin{bmatrix} 1 & 0 & 1-\sigma_2 \\ 0 & 1 & \sigma_1 - 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

is regular since

$$\operatorname{rank} \begin{bmatrix} 1 & 0 & 1-s_2 \\ 0 & 1 & s_1 - 1 \\ 0 & 0 & 1 \end{bmatrix} = \operatorname{rank} \begin{bmatrix} 1 & 0 & 1-s_2 \\ 0 & 1 & s_1 - 1 \end{bmatrix} + \operatorname{rank} [0 \;\; 0 \;\; 1].$$

Moreover, the associated w-behavior is obviously the zero behavior $\mathcal{B}_w^d$, showing that $\mathcal{B}_w^d$ is regularly implementable from $\mathcal{B}_{(w,c)}$ by interconnection through c.

Consider now the w-behavior $\mathcal{B}_w$ associated with $\mathcal{B}_{(w,c)}$. It is not difficult to check that

$\mathcal{B}_w = \ker[\sigma_1 - 1 \quad \sigma_2 - 1]$. If $\mathcal{B}_w^d$ were implementable from $\mathcal{B}_w$ by full interconnection, the 2D polynomial matrix $[s_1 - 1 \quad s_2 - 1]$ would be completable to a unimodular matrix, which is not the case since it is not a zero-left-prime matrix. Thus we conclude that $\mathcal{B}_w^d$ is not implementable from $\mathcal{B}_w$ by full interconnection, showing that the 1D result does to generalize to nD behaviors. ■

In view of this example, we will try to characterize regular implementability by interconnection through a control variable in terms of conditions on the control variable behavior.

Let $\mathcal{B}_{(w,c)}$ be described by the equation $Rw = Mc$ and $\mathcal{B}_w^d \subset \mathcal{B}_w$. From now on we will make the following asssumption.

Assumption 4.1. *$\mathcal{B}_w^d$ is implementable from $\mathcal{B}_{(w,c)}$ by interconnection through c.*

Define now the behaviors $\mathcal{B}_c$ as the c-behavior associated with $\mathcal{B}_{(w,c)}$, i.e. $\mathcal{B}_c = \Pi_c(\mathcal{B}_{(w,c)})$, and $\mathcal{B}_c^d := \{c | \exists w : (w,c) \in \mathcal{B}_{(w,c)} \wedge w \in \mathcal{B}_w^d\}$.

Theorem 4.2. *With the previous notation and under Assumption 4.1, the following statements are equivalent.*

1. *$\mathcal{B}_w^d$ is regularly implementable from $\mathcal{B}_{(w,c)}$ by interconnection through c.*

2. *$\mathcal{B}_c^d$ is regularly implementable from $\mathcal{B}_c$ by full interconnection.*

The proof of this result is based on the following lemma.

Lemma 4.1. *Let R, M and K be nD polynomial matrices such that R and M have the same number of rows and M and C have the same number of columns. Let further N be a MLA of R. Then*

$$\operatorname{rank} \begin{bmatrix} R & -M \\ 0 & K \end{bmatrix} = \operatorname{rank} [R \quad -M] + \operatorname{rank} \begin{bmatrix} NM \\ K \end{bmatrix} - \operatorname{rank} NM. \tag{4.8}$$

Proof

Let $G = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}$ be a square nonsingular nD polynomial matrix such that

$$G[R \quad -M] = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} [R \quad -M] = \begin{bmatrix} G_1 R & -G_1 M \\ G_2 R & -G_2 M \end{bmatrix}$$

where $G_1 R$ has full row rank and $G_2 R = 0$. Since G_2 is a left annihilator of R, it can be obtained from the MLA N as $G_2 = XN$, for some suitable nD polynomial matrix X. Now,

$$\operatorname{rank} \begin{bmatrix} R & -M \\ 0 & K \end{bmatrix} = \operatorname{rank} \begin{bmatrix} G & 0 \\ -N & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} R & -M \\ 0 & K \end{bmatrix} = \operatorname{rank} \begin{bmatrix} G_1 R & -G_1 M \\ 0 & XNM \\ 0 & NM \\ 0 & K \end{bmatrix},$$

and therefore, taking into account that G_1R has full row rank, we conclude that

$$\begin{aligned}\operatorname{rank}\begin{bmatrix} R & -M \\ 0 & K \end{bmatrix} &= \operatorname{rank}[G_1R \quad -G_1M] + \operatorname{rank}\begin{bmatrix} 0 & XNM \\ 0 & NM \\ 0 & K \end{bmatrix} \\ &= \operatorname{rank}[G_1R \quad -G_1M] + \operatorname{rank}\begin{bmatrix} NM \\ K \end{bmatrix}. \end{aligned} \tag{4.9}$$

But, on the other hand

$$\begin{aligned}\operatorname{rank}[R \quad -M] &= \operatorname{rank}\begin{bmatrix} G \\ -N \end{bmatrix}[R \quad -M] = \operatorname{rank}\begin{bmatrix} G_1R & -G_1M \\ 0 & XNM \\ 0 & NM \end{bmatrix} \\ &= \operatorname{rank}[G_1R \quad -G_1M] + \operatorname{rank} NM. \end{aligned} \tag{4.10}$$

Combining (4.9) and (4.10) yields (4.8) as desired. ■

Proof of Theorem 4.2

1 ⇒ 2: Assume that condition 1 is satisfied. This means that there exists a nD polynomial matrix K such that $\mathcal{B}_w^d$ is the w-behavior described by

$$\begin{bmatrix} R & -M \\ 0 & K \end{bmatrix}\begin{bmatrix} w \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and moreover

$$\operatorname{rank}\begin{bmatrix} R & -M \\ 0 & K \end{bmatrix} = \operatorname{rank}[R \quad -M] + \operatorname{rank} K. \tag{4.11}$$

Together with (4.8) in Lemma 4.1, (4.11) implies that given a MLA N of R

$$\operatorname{rank}\begin{bmatrix} NM \\ K \end{bmatrix} = \operatorname{rank} NM + \operatorname{rank} K.$$

In other words, $\bar{\mathcal{B}}_c := \ker \begin{bmatrix} NM \\ K \end{bmatrix}$ is regularly implementable from $\mathcal{B}_c = \ker NM$ by full interconnection (with $\mathcal{B}_c^c = \ker K$). Moreover, it is not difficult to check that $\bar{\mathcal{B}}_c \subset \mathcal{B}_c^d (\subset \mathcal{B}_c)$. By [3, Thm 4.2 and Thm 4.5] this implies that $\mathcal{B}_c^d$ is itself implementable from $\mathcal{B}_c$ by full interconnection, proving that condition 2 holds true.

2 ⇒ 1: Assume now that condition 2 is verified. Let $\mathcal{B}_c^c = \ker K$ be such that $\mathcal{B}_c^d$ is the regular full interconnection of $\mathcal{B}_c^c = \ker K$ and $\mathcal{B}_c$. Let further N be a MLA of R. Then $\mathcal{B}_c = \ker NM$ and the regularity of the interconnection $\mathcal{B}_c \cap \mathcal{B}_c^c = \mathcal{B}_c^d$ means that

$$\operatorname{rank}\begin{bmatrix} NM \\ K \end{bmatrix} = \operatorname{rank} NM + \operatorname{rank} K. \tag{4.12}$$

Together with (4.8) in Lemma 4.1, (4.12) implies that

$$\operatorname{rank}\begin{bmatrix} R & -M \\ 0 & K \end{bmatrix} = \operatorname{rank}\,[R \quad -M] + \operatorname{rank} K,$$

meaning that the interconnection of $\mathcal{B}_w$ and $\mathcal{B}_c^c$ through c is regular. Finally, note that $\mathcal{B}_w^d$ is exactly the behavior resulting from that interconnection, proving that condition 1 holds true. ■

Example 4.2. Consider again the behaviors $\mathcal{B}_{(w,c)}$ and $\mathcal{B}_w^d = \{0\}$ of Example 4.1. Then it is clear that c a free variable in $\mathcal{B}_{(w,c)}$, i.e. $\mathcal{B}_c = \ker 0$, and that $\mathcal{B}_c^d = \ker \begin{bmatrix} \sigma_2 - 1 \\ 1 - \sigma_1 \end{bmatrix}$. Since the (trivial) interconnection $\mathcal{B}_c^d = \mathcal{B}_c \cap \mathcal{B}_c^d$ is regular, $\mathcal{B}_c^d$ is regularly implementable from $\mathcal{B}_c$ by full interconnection. This implies that $\mathcal{B}_w^d$ is regularly implementable from $\mathcal{B}_{(w,c)}$ by interconnection through c (which was an already known fact from Example 4.1). Note that in this case, apart from the regular controller $\mathcal{B}_c^c = \ker 1$ given in Example 4.1, we can also take $\ker \begin{bmatrix} \sigma_2 - 1 \\ 1 - \sigma_1 \end{bmatrix}$ as regular controller. This latter is the controller obtained by applying the arguments in the proof of Theorem 4.2. ■

It turns out that condition 2 of Theorem 4.2 (regular implementability of $\mathcal{B}_c^d$ from $\mathcal{B}_c$ by full interconnection) can be checked by means of algorithms based on the (kernel) representations of the given behaviors (see, for instance, [3, Section 3.1]). Thus we will next investigate how such representations can be determined.

Under Assumption 4.1 it follows from Theorem 3.1 that

$$\mathcal{B}_w^h \subset \mathcal{B}_w^d \subset \mathcal{B}_w \tag{4.13}$$

and, as mentioned in the proof of that theorem, there exists a nD polynomial matrix F such that

$$\mathcal{B}_w^d = \ker FR. \tag{4.14}$$

Analogously, the second inclusion in (4.13) implies that there exists a nD polynomial matrix Γ such that

$$\mathcal{B}_w = \ker \Gamma FR. \tag{4.15}$$

Lemma 4.2. *With the previous notation and under Assumption 4.1:*

1. $\mathcal{B}_c = \ker \Gamma FM$
2. $\mathcal{B}_c^d = \ker FM$.

Proof
1: Note that $\{c \in \mathcal{B}_c\} \Leftrightarrow \{\exists\ w : Rw = Mc\} \Leftrightarrow \{\exists\ w \in \mathcal{B}_w : Rw = Mc\} \Leftrightarrow \{\exists\ w : \Gamma FR = 0 \wedge Rw = Mc\}$. Thus, $\mathcal{B}_c$ is the c-behavior described by the equation

$$\begin{bmatrix} R \\ \Gamma FR \end{bmatrix} w = \begin{bmatrix} M \\ 0 \end{bmatrix} c. \tag{4.16}$$

Eliminating the variable w by premultiplying both sides of (4.16) by $[\Gamma F \quad -I]$ (which is a MLA of $\begin{bmatrix} R \\ \Gamma FR \end{bmatrix}$) we obtain that $\mathcal{B}_c = \ker[\Gamma F \quad -I] \begin{bmatrix} M \\ 0 \end{bmatrix} = \ker \Gamma FM$.
2: This statement follows by the same arguments as in 1 from the fact that $\mathcal{B}_c^d$ is the c-behavior described by the equations

$$\begin{bmatrix} R \\ FR \end{bmatrix} w = \begin{bmatrix} M \\ 0 \end{bmatrix} c. \tag{4.17}$$

■

Note that the kernel representations that are originally given for the behaviors $\mathcal{B}_w^d$ and $\mathcal{B}_w$ may differ from (4.14) and (4.15). However, it is not difficult to obtain suitable matrices F and Γ starting from any other representations S^d and S of $\mathcal{B}_w^d$ and $\mathcal{B}_w$ respectively. This allows to construct the matrices FM and ΓFM, and then apply the aforementioned algorithm in [3, Thm 4.2 and Thm 4.5] to test condition 2 of Theorem 4.2.

5 Conclusion

In this paper we have investigated the implementability and regular implementability of a nD behavior $\mathcal{B}_w^d$ (corresponding to control objective) from a given behavior $\mathcal{B}_{(w,c)}$ by means of interconnection through the control variable c.

It turns out that implementability has a similar characterization as in the 1D case. However, the situation is diferent for regular implementability. Indeed, whereas in the 1D case this is equivalent to the regular implementability of $\mathcal{B}_w^d$ from $\mathcal{B}_w = \Pi_w(\mathcal{B}_{(w,c)})$ by full interconnection, the same does not happen in the nD case. Nevertheless, in that case it is still possible to show that that property is equivalent to the regular implementability of $\mathcal{B}_c^d$ from $\mathcal{B}_c$ by full interconnection.

In view of the obtained results, it would be interesting: (i) to know whether in the nD case regular implementability by interconnection through a control variable is an intrinsic property of the system variable behaviors $\mathcal{B}_w^d$ and $\mathcal{B}_w$ and (ii) to obtain the corresponding characterization. These are questions under current investigation.

References

[1] M.N Belur and H.L. Trentelman. *Stabilization, pole placement and regular implementability*, IEEE Trans. Aut. Contr., Vol. 47 (2002), pp. 735-744.

[2] J. Polderman and I. Mareels. *A behavioral approach to adaptive control*, in the Mathematics of Systems and Control: from Intelligent Control to Behavioral Systems, J. Polderman and H. Trentelman, eds., The Netherlands, 1999, Foundation Systems and Control, Groningen, pp. 119-130.

[3] P. Rocha and J. Wood. *Trajectory control and interconnection of 1D and nD systems*, SIAM J. Cont. and Optim., 40 (1), 2001, pp. 107-134.

[4] H. Trentelman. *A truly behavioral approach to the* H_∞ *control problem*, in the Mathematics of Systems and Control: from Intelligent Control to Behavioral Systems, J. Polderman and H. Trentelman, eds., The Netherlands, 1999, Foundation Systems and Control, Groningen, pp. 177-190.

[5] H. Trentelman and J.C. Willems. *H-infinity control in a behavioral context: the full information case*, IEEE Trans. Aut. Contr., Vol. 44 (1999), pp. 521-536.

[6] H. Trentelman and J.C. Willems. *Synthesis of dissipative systems using quadratic differential forms, Part I*, IEEE Trans. Aut. Contr., Vol. 47 (2002), pp. 53 - 69.

[7] J.C. Willems. *Paradigms and puzzles in the theory of dynamical systems*, IEEE Trans. Aut. Contr., Vol. 36 (1991), pp. 259-294.