RESOURCE ALLOCATION AND CONGESTION CONTROL IN DISTRIBUTED SENSOR NETWORKS—A NETWORK CALCULUS APPROACH

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Abstract- The establishment of the overall objectives of a distributed sensor network is a dynamic task so that it may sufficiently well 'track' its environment. Both resource allocation to each input data flow and congestion control at each decision node of such a network must be performed in an integrated framework such that they are sensitive to this dynamically established overall objectives. In this paper, the effectiveness of a 'per-flow' virtual queuing framework that decouples the input data flows to each decision node is demonstrated. Under this framework, the buffer setpoint level of a decision node is established via the control of setpoint levels of individual virtual buffers assigned to each source node. Network calculus notions are utilized to model the end-to-end flow and design a simple yet effective feedback control law for each input data flow. The control strategy, while enabling satisfactory tracking of a dynamically allocated buffer queue setpoint, is also robust against the time-varying nature of network delays and buffer depletion rate.

I INTRODUCTION

Distributed sensor networks (DSNs) utilize a variety of sensors that may be distributed logically, spatially, and geographically. In a highly dynamic environment, the observed 'scene,' and hence the overall objectives, can change frequently and hence a DSN requires a resource allocation and congestion control scheme that is sensitive to the overall objectives of the DSN and accounts for the following concerns in an effective manner [7, 11, 12]:

(a) time-varying (TV) network-induced delays;

(b) inherent nonlinearities of the network such as buffer cutoff/saturation and limitations on the output rate of each node;

(c) dynamic allocation of available bandwidth to input data flows; and

(d) maintenance of satisfactory buffer occupancy levels at each decision node.

Strategies that address some of these concerns have appeared in the literature. For example, effective congestion control strategies based on conventional control theoretic techniques are in [10, 8]. However, in a highly dynamic environment, all these concerns must be addressed in an integrated framework in order to obtain satisfactory results.

The modeling and congestion controller design in the work presented herein utilize notions from network calculus which is an alternate strategy for deterministic analysis of networks. It uses the concept of an impulse response in a certain min-plus algebra to characterize each network element thus providing a convenient mathematical framework for maintaining QoS guarantees [5]. In this paper, we extend and adapt these notions for the purpose of resource allocation and congestion control of DSNs operating in a highly dynamic environment. In particular, the TV nature of network delays, source-node rate cutoff, and dynamic allocation of buffer setpoints and depletion rates are all accounted for. Robustness against these constitute the major significance of the proposed control strategy. The simulation results demonstrate its effectiveness in satisfactorily maintaining the buffer levels.

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This paper is organized as follows: in Section II, a new 'per-flow' virtual queuing framework for analysis of DSNs is presented; in Section III, the main notions and results of min-plus algebra and network calculus needed for the purpose at hand are presented; in Section IV, a new controller strategy is derived using network calculus approach; in Section V, simulation results are provided to justify and clarify the notions presented.

II VIRTUAL PER-FLOW FRAMEWORK

A DSN CONFIGURATION

For simplicity and ease of presentation, consider a hierarchically organized DSN with the following node layers [12]: *sensor-nodes* at the lowest level leaf-node layer; *sup-nodes* at each intermediate level supervisory layers; and the *root node* at the highest level. See Fig. 1. Nodes at a given hierarchical level may com-

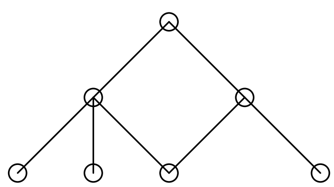

Figure 1: A tree structured DSN.

municate with multiple sup-nodes at the next level. This ensures fault tolerance and adaptability. Information received at each sup-node from its corresponding sensor-nodes must be buffered for processing and extraction of information at a higher abstraction level so that it can then be passed onto the next hierarchical level. This is one major difference between a DSN and a conventional communication network where the primary objective is reliable transfer of information from a source to its destination. Each sup-node of a DSN has a processing bandwidth that is a reflection of its data processing speed. This latter quantity can be looked on as the depletion rate of the sup-node buffer. Clearly, to prevent data loss and improve performance, an effective resource management and congestion control scheme is necessary.

B A VIRTUAL PER-FLOW QUEUING FRAMEWORK

In a highly dynamic environment, the relevance of data from each sensor-node, as perceived by its sup-nodes, the quality of data from each sensor-node, and indeed the number of sensor-nodes in the network are all highly TV. Hence a mechanism that allows nodes at one hierarchical level to allocate system resources dynamically and treat each sensor-node decoupled from the others would be a convenient analytical tool. The 'per-flow' virtual queuing framework in Fig. 2 fulfills these requirements thus rendering the controller design simpler. Here, a 'virtual' buffer (which is not a phys-

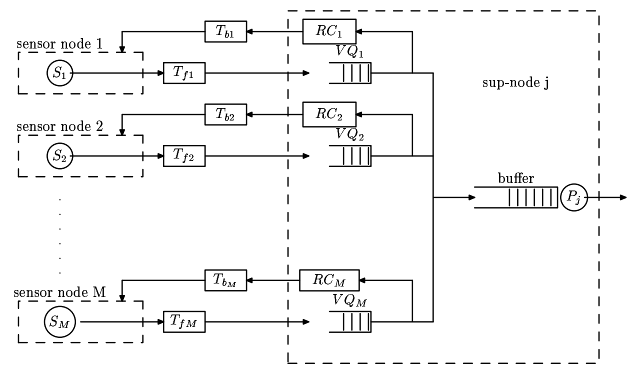

Figure 2: 'Per-flow' virtual queuing framework.

ical entity) is assigned to each virtual circuit or flow. Data cells from each flow gets queued up in its own virtual buffer. Fig. 2 illustrates the situation for M flows; for $i = \overline{1, M}$, S_i are sensor-nodes, T_{f_i} are forward delays, T_{b_i} are backward delays, RC_i are rate controllers (located at the sup-node) that dictate the instantaneous rate for each sensor-node, and VQ_i are virtual buffers. The advantages offered by the per-flow framework in Fig. 2 are the following: (a) system resource allocation algorithm can be seamlessly incorporated by distributing available system bandwidth and buffer level setpoint to the sensor-nodes; (b) the decoupling of the flows makes the analysis easier; (c) it is easier to incorporate different delay characteristics and nonlinearities that may reside in each loop; and (d) it can address end-to-end congestion management.

As will be demonstrated later, this per-flow framework allows setpoint control of the sup-node buffer by controlling the buffer level of each of its virtual buffers. It is our contention that the bandwidth and virtual buffer setpoint the sup-node allocates to each flow must be determined by how important it perceives the information from each sensor-node is. For example, the work in [12] utilize a suitably defined *importance measure* for each flow and allocates virtual buffer setpoints and depletion rates proportionally. This essentially means that, even if the sup-node buffer setpoint and depletion rate are constant, the resources allocated to each virtual buffer can change significantly especially when the DSN is operating in a highly dynamic environment where the importance of sensor-nodes are highly dynamic as well.

III NETWORK CALCULUS

Network calculus is an elegant framework for analysis and maintenance of deterministic QoS guarantees in packet switched networks [5, 9, 4].

A NOTATION

Let $\Re^+$ and $\Re^+_{\min}$ denote the nonnegative reals and $\Re^+ \cup \infty$ respectively. We use $\wedge$ to denote the min-operator. Define

$$\begin{aligned} \Gamma &= \left\{ F : \Re^+ \mapsto \Re^+_{\min} : t \mapsto F(t) \text{ s.t. } F(t) \geq 0 \right\}; \\ \tilde{\Gamma} &= \left\{ \tilde{F} : \Re^+ \times \Re^+ \mapsto \Re^+_{\min} : (s,t) \mapsto \tilde{F}(s,t) \text{ s.t.} \right. \\ &\qquad\qquad \left. F(s,t) \geq 0,\ \forall s \in [0,t] \right\}. \end{aligned} \tag{1}$$

One may view $\tilde{\Gamma}$ as a generalization of Γ via the mapping

$$\begin{aligned} \Gamma \mapsto \tilde{\Gamma} : F(t) &\mapsto \tilde{F}(s,t) \\ &= \begin{cases} F(t-s), & \text{for } s \in [0,t]; \\ \infty, & \text{otherwise.} \end{cases} \end{aligned} \tag{2}$$

When $F(0) = 0$, Γ is denoted as Γ_0; when $\tilde{F}(t,t) = 0,\ \forall t$, $\tilde{\Gamma}$ is denoted as $\tilde{\Gamma}_0$. Given $F \in \Gamma$ (or Γ_0), the function obtained via (2) will be denoted by $\tilde{F} \in \tilde{\Gamma}$ (or $\tilde{\Gamma}_0$). As an example, consider $E \in \Gamma_0$ and $\tilde{E} \in \tilde{\Gamma}_0$ where

$$\begin{aligned} E(t) &= \begin{cases} 0, & \text{for } t = 0; \\ \infty, & \text{otherwise} \end{cases}; \\ \tilde{E}(s,t) &= \begin{cases} E(t-s), & \text{for } s \in [0,t]; \\ \infty, & \text{otherwise.} \end{cases} \end{aligned} \tag{3}$$

B MIN-PLUS ALGEBRA

Definition 1 (Min-plus linearity) *[3] The system operator $\mathcal{S} : \Gamma \mapsto \Gamma : A \mapsto D$ is said to be* (min-plus) linear *if, (i) for any index set $\mathcal{I}$, $\mathcal{S}\left[\min_{i\in\mathcal{I}}\{A_i\}\right] = \min_{i\in\mathcal{I}}\{\mathcal{S}[A_i]\}$, $\forall A_i \in \Gamma$, and (ii) $\mathcal{S}[a + A] = a + \mathcal{S}[A]$, $\forall a \in \Re^+_{\min}$, $\forall A \in \Gamma$.*

Definition 2 (Min-plus convolution) Min-plus convolution (in $(\Gamma, \tilde{\Gamma})$) *of $A \in \Gamma$ and $\tilde{H} \in \tilde{\Gamma}$ is defined as*

$$(A \star \tilde{H})(t) = \min_{s\in[0,t]}\{A(s) + \tilde{H}(s,t)\}.$$

Remarks:

1. For $A \in \Gamma$ and $\tilde{G}, \tilde{H} \in \tilde{\Gamma}$, it is easy to show that
$$\begin{aligned} &((A \star \tilde{G}) \star \tilde{H})(t) \\ &= \min_{s\in[0,t]}\left\{A(s) + \min_{\tau\in[s,t]}\left\{\tilde{G}(s,\tau) + \tilde{H}(\tau,t)\right\}\right\} \end{aligned}$$

2. Hence min-plus convolution (in $(\tilde{\Gamma}, \tilde{\Gamma})$) of $\tilde{G}, \tilde{H} \in \tilde{\Gamma}$ is taken as
$$(\tilde{G} \star \tilde{H})(s,t) \equiv \min_{\tau\in[s,t]}\left\{\tilde{G}(s,\tau) + \tilde{H}(\tau,t)\right\}.$$
Now the relationship $(A \star \tilde{G}) \star \tilde{H} = A \star (\tilde{G} \star \tilde{H})$ holds.

3. With Definition 2 in mind, and using the mapping (2), min-plus convolution (in (Γ, Γ)) for $A, B \in \Gamma$ may be defined as
$$(A \star B)(t) \equiv \min_{s\in[0,t]}\{A(s) + B(t-s)\}.$$

4. We use $A^{(i)}$ to denote the i-fold min-plus convolution of A with itself.

5. Elements of Γ_0 and $\tilde{\Gamma}_0$ are referred to as *processes* [5, 4]. Flows in a network, when they represent the number of cells accumulated during the time interval $[0,t]$, can be described via processes.

Min-plus convolution forms the basis for the following notions:

Theorem 1 (Impulse response) *[3] For a given linear system $\mathcal{S}[\cdot]$, there exists a unique function $\tilde{H} \in \tilde{\Gamma}$, referred to as its* impulse response, *s.t., for all input-output pairs $\{A, D\},\ A, D \in \Gamma$,*

$$D(t) = \mathcal{S}[A(t)] = \min_{s \in \Re^+} \left\{ A(s) + \tilde{H}(s,t) \right\}.$$

Unless otherwise mentioned, we deal with causal systems for which Theorem 1 yields

$$D(t) = \mathcal{S}[A(t)] = (A \star \tilde{H})(t).$$

The following result will be useful later.

Lemma 1 *Consider the input process $\in \Gamma_0$ to a buffer which possesses a depletion rate $b(t)$ and depletion process $B(t) = \sum_{\tau=0}^{t} b(\tau) \in \Gamma_0$. Then the corresponding output is $D(t) = (A \star \tilde{S}_B)(t) \in \Gamma_0$ where*

$$\tilde{S}_B(s,t) = B(t) - B(s) \in \tilde{\Gamma}_0.$$

Proof: For causality, we clearly must have

$$D(t) \leq A(s) + B(t) - B(s),\ \forall s \in [0,t].$$

This, together with the fact that $D(t) = B(t),\ \forall t \in \Re^+$, yield

$$D(t) = \min_{s \in [0,t]} \left\{ A(s) + B(t) - B(s) \right\}.$$

This proves the claim. •

Remark: Lemma 1 imply that, for input processes in Γ_0, $\tilde{S}_B \in \tilde{\Gamma}_0$ can be considered the 'impulse response' of a buffer with depletion process $B \in \Gamma_0$.

Definition 3 *The flow $\tilde{R} \in \tilde{\Gamma}$ (or $R \in \Gamma$) is said to be* $\tilde{\Lambda}$-constrained *if $\tilde{R}(s,t) \leq (\tilde{R} \star \tilde{\Lambda})(s,t),\ \forall s \in [0,t]$ (or $R(t) \leq (R \star \tilde{\Lambda})(s,t),\ \forall s \in [0,t]$).*

The above definitions provide the following important result:

Theorem 2 (Backlog bound theorem) *Consider a $\tilde{\Lambda}$-cons-trained, but otherwise arbitrary, input $A \in \Gamma$ to a network element with impulse response $\tilde{S} \in \tilde{\Gamma}$. Then the* backlog *of the network element defined as $A(t) - D(t)$ is guaranteed to be bounded by $Q \in \Gamma$ if $\tilde{\Lambda}(s,t) \leq \tilde{S}(s,t) + Q(t),\ \forall s \in [0,t]$.*

Proof: Note that $A(t) \leq (A \star \tilde{\Lambda})(t)$ and $\tilde{\Lambda}(s,t) \leq \tilde{S}(s,t) + Q(t),\ \forall s \in [0,t]$, imply that

$$\begin{aligned} A(t) &\leq \min_{s \in [0,t]} \left\{ A(s) + \tilde{\Lambda}(s,t) \right\} \\ &\leq \min_{s \in [0,t]} \left\{ A(s) + \tilde{S}(s,t) + Q(t) \right\} \\ &= \min_{s \in [0,t]} \left\{ A(s) + \tilde{S}(s,t) \right\} + Q(t) \\ &= D(t) + Q(t), \end{aligned}$$

where we used the relationship $D(t) = (A \star \tilde{S})(t)$. •

Remark: It is easy to show that, if $\tilde{S} \in \tilde{\Gamma}_0$, the equality in the proof above will be achieved, viz., the backlog can be made *equal* (not only bounded) to $Q \in \Gamma$.

Definition 4 (Closure) *[3] The* closure operator $\mathcal{S}^*$ *of the operator $\mathcal{S}$ is $\mathcal{S}^*(\cdot) = \inf\{\cdot, \mathcal{S}(\cdot), \mathcal{S}^{(2)}(\cdot), \dots\}$; the closure $\tilde{F}^* \in \tilde{\Gamma}_0$ of a function $\tilde{F} \in \tilde{\Gamma}$ is $\tilde{F}^* = \inf\{\tilde{E}, \tilde{F}, \tilde{F}^{(2)}, \dots\}$, where $\tilde{E}(s,t) \in \tilde{\Gamma}_0$ is as in (3).*

Remark: Note that the closure is necessarily an element of $\tilde{\Gamma}_0$.

Definition 5 (Subadditivity) *A function $\tilde{G} \in \tilde{\Gamma}$ is said to be* subadditive *if*

$$\tilde{G}(s,t) \leq \tilde{G}(s,\tau) + \tilde{G}(\tau,t),\ \forall \tau \in [s,t].$$

The following properties can be easily established:

Theorem 3

(i) For arbitrary $\tilde{G} \in \tilde{\Gamma}$, $\tilde{G}^*$ is subadditive.

(ii) If $\tilde{G} \in \tilde{\Gamma}$ is subadditive, then $\tilde{G} \star \tilde{G} = \tilde{G}$ and $\tilde{G}^* = \tilde{G}$.

Theorem 4 *If $F \in \Gamma$ is $\tilde{\Lambda}^*$-constrained, then it is $\tilde{\Lambda}$-cons-trained as well.*

Proof: Since F is $\tilde{\Lambda}^*$-constrained, we have $F(t) \leq (F \star \tilde{\Lambda}^*)(t)$, viz.,

$$F(t) \leq \min_{s \in [0,t]} \left\{ F(s) + \tilde{\Lambda}^*(s,t) \right\}.$$

Now use the fact that $\tilde{\Lambda}^* \leq \tilde{\Lambda},\ \forall (s,t)$:

$$F(t) \leq \min_{s \in [0,t]} \left\{ F(s) + \tilde{\Lambda}(s,t) \right\}.$$

Hence F is $\tilde{\Lambda}$-constrained as well. •

Theorem 5 (Departure process constraint) *The output of a linear network element with impulse response $\tilde{\Lambda}^* \in \tilde{\Gamma}_0$ is $\tilde{\Lambda}$-constrained.*

Proof: Let $A, D \in \Gamma$ denote the input and output of the network element respectively. Then

$$D \star \tilde{\Lambda}^* = A \star \tilde{\Lambda}^* \star \tilde{\Lambda}^* = A \star \tilde{\Lambda}^* = D.$$

Hence D in $\tilde{\Lambda}^*$-constrained. Now apply Theorem 4 to prove the claim. •

IV VIRTUAL PER-FLOW CONGESTION CONTROL

A IMPULSE RESPONSES OF NETWORK DELAY ELEMENTS

Perhaps the first study of the types of TV delays that may occur in a network and their system theoretic models in a discrete-time setting appeared in [6]. Further work appear in [11] and [7]; a continuous-time analysis is in [2].

The underlying 'sampling unit time' is denoted by Δ. This corresponds to the rate at which the sensor-node rate is computed by the discrete-time controller. All delays are therefore nonnegative integer multiples of Δ.

A.1 FORWARD DELAY ELEMENT

Forward delay $T_f(t)$ is experienced by the cell flow from a sensor-node to a sup-node (see Fig. 2). If $A, D \in \Gamma_0$ denote, respectively, the accumulated number of cells arriving and leaving the forward delay element during the time interval $[0, t]$, the input/output relationship of a forward delay element can be described via

$$D(t) = A(t - T_f(t)),$$

where the characteristics of $T_f(t)$ (in particular, restrictions on how it may vary from one time instant to the other) are described in detail in [6, 7, 11]. We do not provide these details here because they are not very crucial for the results presented in this paper. On the other hand, what is important realize is that, from Theorem 1, the impulse response of the forward delay element can be found as

$$\tilde{S}_{T_f}(s,t) = \begin{cases} 0, & \text{if } t - s = T_f(t); \\ \infty, & \text{otherwise.} \end{cases} \qquad (4)$$

Observe that $\tilde{S}_{T_f} \in \tilde{\Gamma}$ and $D(t) = (A \star \tilde{S}_{T_f})(t)$. The minimum and maximum possible values of the forward delay are denoted by $\underline{T}_f$ and $\overline{T}_f$ respectively.

A.2 BACKWARD DELAY ELEMENT

We assume that feedback information, sent from a sup-node to a sensor-node, is carried by control cells that are periodically (in terms of the number of cells) inserted into the data stream[1]. The backward delay is hence experienced by control cells only (see Fig. 2). In this paper, what is being fed back is taken to be the process information, viz., the information carried by control cells indicate the accumulated number of cells the virtual controller wants the sensor-node to have sent within the duration $[0, t]$. This is a departure from the usual practice where the control information is the desired sensor-node rate. Although either is a simply a matter of choice, the approach taken in this paper allows us to model the control flow and other related flows as elements of Γ which requires nonnegativity (see (1)). As before, if $A, D \in \Gamma_0$ denote, respectively, the accumulated number of cells arriving and leaving the backward delay element during the time interval $[0, t]$, its input/output relationship can be described via

$$D(t) = A(t - T_b(t)),$$

where $T_b(t)$ denotes the time difference between the time instance the control information is ready and the instance it arrives at the sensor-node. We assume that, if no control cell is received during $(t-1, t]$, the sensor-node will assume congestion in the data flow path and stop sending data until it receives a new control cell. Hence, the characteristics of $T_b(t)$ are identical to those of $T_f(t)$ [6, 7, 11]. As before, the impulse response of the backward delay element is

$$\tilde{S}_{T_b}(s,t) = \begin{cases} 0, & \text{if } t - s = T_b(t); \\ \infty, & \text{otherwise.} \end{cases} \qquad (5)$$

Observe that $\tilde{S}_{T_b} \in \tilde{\Gamma}$ and $D(t) = (A \star \tilde{S}_{T_b})(t)$. The minimum and maximum possible values of the backward delay are denoted by $\underline{T}_b$ and $\overline{T}_b$ respectively.

We also identify two types of roundtrip delays:

$$\mathcal{T}_f = T_f(t) + T_b(t - T_f(t));$$
$$\mathcal{T}_b = T_b(t) + T_f(t - T_b(t)).$$

[1] In ATM networks, RM cells play this role.

Also, $\underline{\mathcal{T}} \equiv \underline{T}_f + \underline{T}_b$ and $\overline{\mathcal{T}} \equiv \overline{T}_f + \overline{T}_b$.

A single loop in the virtual framework in Fig. 2 is depicted in Fig. 3; the forward delay, virtual buffer, and backward delay network elements are identified via their impulse responses $\tilde{S}_{T_f} \in \tilde{\Gamma}$, $\tilde{S}_B \in \tilde{\Gamma}_0$, and $\tilde{S}_{T_b} \in \tilde{\Gamma}$ respectively.

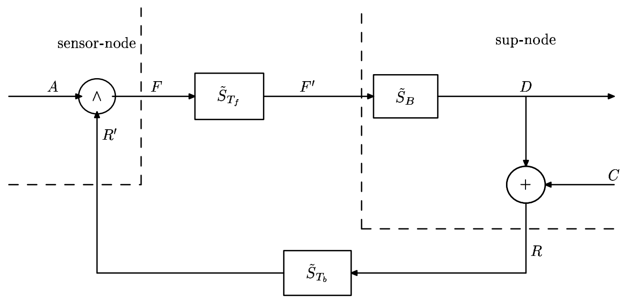

Figure 3: A single loop of the per-flow framework.

In Fig. 3, $A \in \Gamma_0$ is the input cell flow process of the sensor-node, $F, F' \in \Gamma_0$ are the data cell flow processes from sensor-node to sup-node before and after experiencing the corresponding forward delay, and $D \in \Gamma_0$ is the output cell flow process of the virtual buffer at sup-node. Of course, this latter process is identical to the depletion process $B \in \Gamma_0$ of the virtual buffer. The control information is carried in C, R, and R'. These are necessarily *processes;* $R, R' \in \Gamma$ are the control cell flows from sup-node to sensor-node before and after experiencing the corresponding backward delay, and $C \in \Gamma$ is the control command which, when added to the virtual buffer output process $D \in \Gamma_0$, indicates accumulated number of cells the virtual controller wants the sensor-node to have sent within the duration $[0, t]$.

B CONTROL OBJECTIVE

Our control objective is to ensure that the virtual buffer backlog $F' - D = F' - B$ remains at a dynamically assigned setpoint level $Q \in \Gamma$. With the impulse response $\tilde{S}_B \in \tilde{\Gamma}_0$ of the virtual buffer and Theorem 2 in hand, we may ensure this if the arrival process $F' \in \Gamma_0$ can be constrained by $\tilde{\Lambda}$ where $\tilde{\Lambda}(s,t) \leq \tilde{S}_B(s,t) + Q(t),\ \forall s \in [0,t]$. To proceed, we first need the impulse response of the throttle node (indicated by $\wedge$ in Fig. 3).

B.1 IMPULSE RESPONSE OF THROTTLE NODE

Note that

$$F' = F \star \tilde{S}_{T_f}; \qquad D = F' \star \tilde{S}_B;$$
$$R = D + C = D \star \tilde{H}; \quad R' = R \star \tilde{S}_{T_b},$$

where $\tilde{H} \in \tilde{\Gamma}$ is given by

$$\tilde{H}(s,t) = \begin{cases} C(t), & \text{for } s = t; \\ \infty, & \text{otherwise.} \end{cases}$$

Hence

$$\begin{aligned} R' &= (F \star \tilde{S}_{T_f} \star \tilde{S}_B + C) \star \tilde{S}_{T_b} \\ &= F \star \tilde{S}_{T_f} \star \tilde{S}_B \star \tilde{H} \star S_{T_b} = F \star \tilde{S}_{\text{loop}}, \end{aligned}$$

where $\tilde{S}_{\text{loop}} \equiv \tilde{S}_{T_f} \star \tilde{S}_B \star \tilde{H} \star \tilde{S}_{T_b} \in \tilde{\Gamma}$. Then the output process of the sensor-node is

$$F = R' \wedge A = (F \star \tilde{S}_{\text{loop}}) \wedge A. \tag{6}$$

The solution to this can be obtained via the following result:

Lemma 2 *If*

$$C(t) > B(t + \overline{\mathcal{T}}) - B(t),\ \forall t \geq -\overline{T}_b, \tag{7}$$

then $F = A \star \tilde{S}^*_{\text{loop}}$ *is the unique solution of (6) and hence* $\tilde{S}^*_{\text{loop}}$ *is the impulse response of the throttle node.*

Proof: Use Lemma 8 of [1]: if

$$\inf_{u,v: s \leq u \leq v \leq t} \tilde{S}_{\text{loop}}(u,v) > 0,\ \forall s \in [0,t], \tag{8}$$

then $A \star \tilde{S}^*_{loop}$ is the unique solution of (6). Let us proceed as follows:

$$\begin{aligned} (\tilde{S}_{T_f} \star \tilde{S}_B)(s,t) &= \min_{\tau \in [s,t]} \left\{ \tilde{S}_{T_f}(s,\tau) + \tilde{S}_B(\tau,t) \right\} \\ &= \tilde{S}_B(s + T_f(f(s)), t) \\ &= B(t) - B(s + T_f(f(s))), \end{aligned}$$

where $f(s)$ is the maximum solution of the equation $\tau - T_f(\tau) = s$ for a given $s \in [0,t]$. In arriving at this, we have used (4) and the fact that $B \in \Gamma_0$ is nondecreasing. Now it is fairly straight-forward to obtain an expression for $\tilde{S}_{\text{loop}}$:

$$\begin{aligned}\tilde{S}_{\text{loop}} &= (\tilde{S}_{T_f} \star \tilde{S}_B) \star \tilde{S}_{T_b} + C \star \tilde{S}_{T_b} \\ &= \tilde{S}_B(s + T_f(f(s)), t - T_b(t)) + C(t - T_b(t)) \\ &= B(t - T_b(t)) - B(s + T_f(f(s))) \\ &\qquad + C(t - T_b(t)),\end{aligned} \tag{9}$$

where we have used (5). Hence, to ensure (8), we need

$$C(t - T_b(t)) > B(s + T_f(f(s))) - B(t - T_b(t)),$$

for all $s \in [0, t]$. Again noting that $B \in \Gamma_0$ is nondecreasing, a corresponding sufficient condition is

$$C(t - T_b(t)) > B(t + T_f(f(s))) - B(t - T_b(t)),$$

for all $s \in [0, t]$. Changing the variable $t - T_b(t)$, it is easy to see that (7) is indeed a sufficient condition for this to be true. •

C DERIVATION OF THE CONTROL STRATEGY

As elaborated upon in Section B, if we desire to keep the virtual buffer level bounded by $Q \in \Gamma$, its arrival process $F' \in \Gamma_0$ should be $\tilde{\Lambda}$-constrained where $\tilde{\Lambda}(s,t) \leq \tilde{S}_B(s,t) + Q(t), \ \forall s \in [0,t]$. As Theorem 5 implies, we can ensure this if F' is the output process of a network element with impulse response $\tilde{\Lambda}^*$, viz., $F' = A \star \tilde{\Lambda}^*$. But, we already know that $F' = A \star \tilde{S}^*_{\text{loop}} \star \tilde{S}_{T_f}$. Hence, to implement the control strategy, a sufficient condition is

$$\tilde{\Lambda}^* = \tilde{S}^*_{\text{loop}} \star \tilde{S}_{T_f}. \tag{10}$$

The remainder of this section is dedicated to establishing the conditions required to solve this for $\tilde{\Lambda}$.

C.1 AN UPPER BOUND ON CONTROL FLOW

Lemma 3 *If*

$$C(t) \geq B(t + \overline{\mathcal{T}}) - B(t), \ \forall t \geq -\overline{T}_b, \tag{11}$$

then $\tilde{S}_{\text{loop}}$ is subadditive.

Proof: Use (9) to show the following:

$$\begin{aligned}&\tilde{S}_{\text{loop}}(s,\tau) + \tilde{S}_{\text{loop}}(\tau,t) - \tilde{S}_{\text{loop}}(s,t) \\ &\quad = C(\tau - T_b(\tau)) - B(\tau + T_f(f(\tau))) \\ &\qquad + B(\tau - T_b(\tau)), \text{ for arbitrary } \tau \in [s,t].\end{aligned}$$

Now apply Definition 5: $\tilde{S}_{\text{loop}}$ is subadditive iff, $\forall \tau \in [s,t]$,

$$C(\tau - T_b(\tau)) \geq B(\tau + T_f(f(\tau))) - B(\tau - T_b(\tau)).$$

Change the variable $\tau - T_b(\tau)$ and take into account the fact that $B \in \Gamma_0$ is nondecreasing to establish the claim. •

Lemma 4 *If*

$$C(t) \geq B(t + \overline{\mathcal{T}} + \overline{T}_f) - B(t), \ \forall t \geq -\overline{\mathcal{T}}, \tag{12}$$

then $\tilde{S}^*_{\text{loop}} \star \tilde{S}_{T_f}$ is subadditive.

Proof: Since (12) implies (11), from Lemma 3, $\tilde{S}_{\text{loop}}$ is subadditive, and therefore, $\tilde{S}^*_{\text{loop}} = \tilde{S}_{\text{loop}}$. We then have

$$\tilde{S}^*_{\text{loop}} \star \tilde{S}_{T_f} = \tilde{S}_{\text{loop}} \star \tilde{S}_{T_f} = \tilde{S}_{\text{loop}}(s, t - T_f(t)).$$

Now apply Definition 5: $\tilde{S}^*_{\text{loop}} \star \tilde{S}_{T_f}$ is subadditive if, $\forall \tau \in [s,t]$,

$$C(\tau - \mathcal{T}_f(\tau)) \geq B(\tau + T_f(f(\tau))) - B(\tau - \mathcal{T}_f(\tau)).$$

Change the variable $\tau - \mathcal{T}_f(\tau)$ and take into account the fact that $B \in \Gamma_0$ is nondecreasing to establish the claim. •

In summary, Lemmas 3 and 4 provide conditions on the control flow $C(t)$ so that $\tilde{S}_{\text{loop}}$ and $\tilde{S}^*_{\text{loop}} * \tilde{S}_{T_f}$ are subadditive, respectively. A sufficient condition for both these quantities to be subadditive is (12). The latter condition however may not imply (7) unless we impose the following upper bound on the control flow:

$$C(t) > B(t + \overline{\mathcal{T}} + \overline{T}_f) - B(t), \ \forall t \geq -\overline{\mathcal{T}}. \tag{13}$$

Lemma 5 *Suppose the control flow $C \in \Lambda$ satisfies (13). Then a solution for $\tilde{\Lambda}^* = \tilde{S}^*_{\text{loop}} \star \tilde{S}_{T_f}$ in (10) is $\tilde{\Lambda} = \tilde{S}_{\text{loop}} * \tilde{S}_{T_f}$.*

Proof: Suppose $\tilde{\Lambda} = \tilde{S}_{\text{loop}} * \tilde{S}_{T_f}$. Then

$$\tilde{\Lambda}^* = \left(\tilde{S}_{\text{loop}} * \tilde{S}_{T_f}\right)^* = \left(\tilde{S}^*_{\text{loop}} * \tilde{S}_{T_f}\right)^* = \tilde{S}^*_{\text{loop}} * \tilde{S}_{T_f}.$$

This completes the proof. •

C.2 A LOWER BOUND ON CONTROL FLOW

Now, with (13) true, there remains only one more condition to keep the virtual buffer level at the dynamically assigned setpoint $Q \in \Gamma$:

Lemma 6 *If (13) is true and*

$$C(t) \leq B(t+\underline{\mathcal{T}}) - B(t) + Q(t+\underline{\mathcal{T}}),\ \forall t \geq -\overline{\mathcal{T}}, \quad (14)$$

then $\tilde{\Lambda}(s,t) \leq \tilde{S}_B(s,t) + Q(t),\ \forall s \in [0,t]$.

Proof: Lemma 5 implies that

$$\begin{aligned}\tilde{\Lambda} &= \tilde{S}_{\text{loop}} * \tilde{S}_{T_f} \\ &= \tilde{S}_{T_f} \star \tilde{S}_B \star \tilde{S}_{T_b} \star \tilde{S}_{T_f} + C \star \tilde{S}_{T_b} \star \tilde{S}_{T_f} \\ &= \tilde{S}_{T_f} \star \tilde{S}_\ell + C_\ell,\end{aligned}$$

where $\tilde{S}_\ell \in \tilde{\Gamma}$ and $C_\ell \in \Gamma$ are defined as

$$\begin{aligned}\tilde{S}_\ell(s,t) &\equiv (\tilde{S}_B \star \tilde{S}_{T_b} \star \tilde{S}_{T_f})(s,t) \\ &= \tilde{S}_B(s, t - \mathcal{T}_f(t)) = B(t - \mathcal{T}_f(t)) - B(s); \\ C_\ell(t) &\equiv (C \star \tilde{S}_{T_b} \star \tilde{S}_{T_f})(t) \\ &= C(s, t - \mathcal{T}_f(t)). \end{aligned} \quad (15)$$

We also have

$$\begin{aligned}(\tilde{S}_{T_f} \star \tilde{S}_\ell)(s,t) &= \tilde{S}_B(s + T_f(f(s)), t - \mathcal{T}_f(t)) \\ &= B(t - \mathcal{T}_f(t)) - B(s + T_f(f(s))).\end{aligned}$$

Hence, noting that $B \in \Gamma_0$ is nondecreasing, we have

$$(\tilde{S}_{T_f} \star \tilde{S}_\ell) \leq \tilde{S}_\ell,\ \forall s \in [0,t].$$

Therefore, $\forall s \in [0,t]$,

$$\tilde{\Lambda} = (\tilde{S}_{\text{loop}} \star \tilde{S}_{T_f})(s,t) \leq \tilde{S}_\ell(s,t) + C_\ell(t).$$

Hence, a sufficient condition for $\tilde{\Lambda}(s,t) \leq \tilde{S}_B(s,t) + Q(t), \forall s \in [0,t]$, to be valid is

$$\tilde{S}_\ell(s,t) + C_\ell(t) \leq \tilde{S}_B(s,t) + Q(t),\ \forall s \in [0,t].$$

Substitute from (15):

$$C(t - \mathcal{T}_f(t)) \leq B(t) - B(t - \mathcal{T}_f(t)) + Q(t),\ \forall t \geq 0.$$

Change the variable $t - \mathcal{T}_f(t)$ and take into account the fact that $B \in \Gamma_0$ is nondecreasing to establish the claim. •

C.3 CONTROL STRATEGY

In summary, to implement our control objective of keeping the virtual buffer level at the dynamically established setpoint $Q \in \Gamma$, we combine (13) and (14): For all $t \geq -\overline{\mathcal{T}}$,

$$\begin{aligned}&B(t + \overline{\mathcal{T}} + \overline{T}_f) - B(t) \\ &\quad < C(t) \leq B(t + \underline{\mathcal{T}}) - B(t) + Q(t + \underline{\mathcal{T}}),\end{aligned}$$

Hence the total feedback process $R = C + D = C + B \in \Gamma$ satisfies

$$B(t + \overline{\mathcal{T}} + \overline{T}_f) < R(t) \leq B(t + \underline{\mathcal{T}}) + Q(t + \underline{\mathcal{T}}),\quad \forall t \geq -\overline{\mathcal{T}}. \quad (16)$$

This of course requires the following constraint on the setpoint dynamics:

$$Q(t + \underline{\mathcal{T}}) > B(t + \overline{\mathcal{T}} + \overline{T}_f) - B(t + \underline{\mathcal{T}}),\ \forall t \geq -\overline{\mathcal{T}}. \quad (17)$$

Assuming (17) to be true, suppose we adopt the control strategy

$$R(t) = B(t + \underline{\mathcal{T}}) + Q(t + \underline{\mathcal{T}}),\ \forall t \geq -\overline{\mathcal{T}}.$$

With this strategy, the rate with which the sensor-node is required to send data is

$$\begin{aligned}r(t) &= R(t) - R(t-1) \\ &= b(t + \underline{\mathcal{T}}) + q(t + \underline{\mathcal{T}}),\ \forall t \geq -\overline{\mathcal{T}}, \end{aligned} \quad (18)$$

where $B(t) = \sum_{\tau=0}^{t} b(\tau)$ and $Q(t) = \sum_{\tau=0}^{t} q(\tau)$.

We may study the effect of sensor rate cutoff on this control strategy as follows: Suppose $\underline{r} \leq r(t) \leq \overline{r}$. Then (18) provides bounds on the dynamics of the virtual buffer setpoint that guarantees success in the control objective of keeping the buffer level at the setpoint:

$$\underline{r} - b(t) \leq q(t) \leq \overline{r} - b(t),\ \forall t \geq -\overline{\mathcal{T}} + \underline{\mathcal{T}}. \quad (19)$$

For example, suppose $Q(t)$ is decreased so that $q(t)$ moves beyond the left hand limit of (19). There are essentially two ways the sensor-node can 'accommodate' this: an increased depletion rate of its virtual buffer and/or a decreased rate at which it can transmit data. If the depletion rate is held constant and the sensor rate 'hits' its minimum $\underline{r}$, the control strategy above will fail. The only solution available is to temporarily

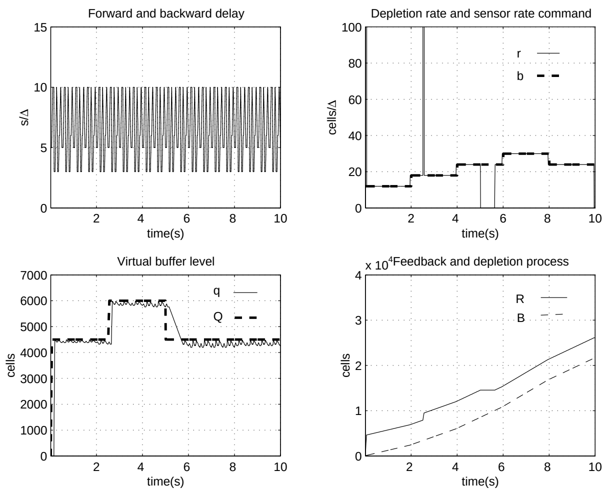

Figure 4: Simulation results.

stop the sensor-node from transmitting altogether; this may be implemented by holding the control command $R(t)$ constant until virtual buffer level goes down to the desired setpoint at the depletion rate currently allocated to it. Once this is achieved, the control law in (16) can be resumed.

In essence, the dynamics of the virtual buffer setpoint should, whenever possible, be confined to (19). When this is true, use the control law in (16):

(a) When (19) is violated from the left hand side, hold the feedback control command constant at the current value until the buffer level reaches the desired setpoint.

(b) When (19) is violated from the right hand side, one runs the risk of an overflowing buffer and data loss.

The above strategy also takes care of a situation when the constraint on setpoint dynamics in (17) may be violated if, for example, the setpoint experiences a significant decrease while the depletion rate is low.

V SIMULATION RESULTS AND CONCLUSION

To illustrate the effectiveness of the proposed control strategy, several simulations were performed. Virtual per-flow framework enables setpoint control of the bottleneck buffer via the setpoint control of each of the virtual buffers. Hence, the controllers may be independently designed for each virtual buffer; Fig. 4 shows the performance of only one such virtual buffer. The major parameters used in this simulation—buffer setpoint level trajectory, TV forward and backward delays, and the TV depletion rate—are shown in Fig. 4. The underlying sampling unit time Δ of the simulation is 10 ms, and as one can observe from Fig. 4, we have used the values

$$\{\underline{T}_f, \overline{T}_f\} = \{\underline{T}_b, \overline{T}_b\} = \{3, 10\};$$
$$\{\underline{T}, \overline{T}\} = \{6, 20\} \text{ units.}$$

We assume that this information is available; in a DSN environment, it is also reasonable to assume that information regarding the depletion rate allocated to each virtual buffer is available to the the virtual controller especially when the latter is located at the sup-node itself where the bandwidth allocation decision is being made [12].

The simulation results clearly illustrate the effectiveness of the proposed control strategy; it maintains tight buffer level control in the presence of TV delays and buffer depletion rate. It is these robustness properties that render the work in this paper significant. In the simulation in Fig. 4, the increase in buffer setpoint command does not violate (19) while its decrease does. Correspondingly, notice how the buffer level faithfully follows the increase, while it is noticeably 'sluggish' at the decrease and how the feedback control flow is held constant until the buffer level 'meets' the setpoint; also note the fact that the sensor rate command is reduced to zero during this time period.

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