2D Linear Control Systems - From Theory to Experiment to Theory

E. Rogers[1], K. Gałkowski[2], D. H. Owens[3], T Al-Towlem[1], J. D. Ratcliffe[1], P. L. Lewin[1]

[1] **Department of Electronics and Computer Science, University of Southampton, Southampton SO17 1BJ, UK.** etar@ecs.soton.ac.uk

[2] **Institute of Control and Computation Engineering, University of Zielona Góra, Poland.**

3 **Department of Automatic Control and Systems Engineering, University of Sheffield, Sheffield S1 3JD, UK.**

Abstract

The subject area of this paper is the application of systems theory developed for linear repetitive processes, a distinct class of 2D linear systems, to linear iterative learning control schemes. A unique feature is the inclusion of experimental results obtained from the application of control laws designed using this theory to an experimental rig in the form of a chain conveyor system. Some areas for further theoretical research arising from this study are also briefly discussed.

1 Introduction

Iterative learning control, denoted by ILC from this point onwards, is a technique to control systems operating in a repetitive mode with the additional requirement that a specified output trajectory $r(t)$ over a finite interval $[0, T]$ is followed to a high precision. Examples of such systems include robotic manipulators that are required to repeat a given task to a high precision, chemical batch processes or, more generally, the class of tracking systems. Motivated by human learning, the basic idea of ILC is to use information from previous executions of the task in order to improve performance from trial to trial in the sense that the tracking error is sequentially reduced. The objective of ILC is to use the repetitive nature of the process to progressively improve the accuracy with which the operation is achieved by updating the control input iteratively from trial to trial.

Since the original work by Arimoto et al [3], the general area of ILC has been the subject of intense research effort both in terms of the underlying theory and 'real world' applications. One starting point for the literature here is the text [6] which gives a good survey up to 1992. For the recent 'state of the art' see, for example, [7].

Typical ILC algorithms construct the input to the plant on a given trial from the input used on the last trial plus an additive increment which is typically a function of the past values of the observed output error, i.e. the difference between the achieved and desired plant output. Suppose that $u_k(t)$ denotes the input to the plant on trial k and that the (finite and constant) trial length is denoted by T, i.e. $t \in [0, T]$. Suppose also that $e_k(t)$ denotes the difference between the desired trajectory $r(t)$ and the system output $y_k(t)$ on the same trial.

Then the objective of constructing a sequence of input functions such that the performance is gradually improving with each successive trial can be refined to a convergence condition on the input and error:

$$\lim_{k\to\infty} ||e_k|| = 0, \; \lim_{k\to\infty} ||u_k - u_\infty|| = 0 \tag{1.1}$$

where $||\cdot||$ is a signal norm in a suitably chosen function space (e.g., $L_2^m[0,T]$) with a norm-based topology.

This definition of convergent learning is, in effect, a stability problem on an infinite-dimensional two-dimensional (2D)-product space, typically of the form $\mathbb{N} \times L_2[0,T]$. As such, it places the analysis of ILC schemes firmly outside standard (termed 1D here) control theory – although (as we will see later in this paper) it still has an important role to play in certain cases of strong practical interest. Instead, ILC schemes must be seen in the context of fixed–point problems or, more precisely, repetitive processes [10].

In general, the work reported to date on ILC schemes can be divided (at a general level) into two classes. In the larger class, nonlinear systems and, in particular, practically motivated model structures typically encountered in robotics or process industries applications are studied. A key point here is that (in the main) the results currently available typically make critical use of special structural properties of the models of the underlying dynamics, e.g., those arising in control oriented models of mechanical or electro–mechanical systems.

This paper focuses on the second class where linear systems are considered in a general setting. One immediate consequence of this choice is that well known analysis and design concepts are theoretically available for use (with appropriate modifications as necessary). For example, frequency domain methods have been used to derive convergence criteria in the form of spectral conditions on the transfer function matrix describing the underlying (pass-to-pass) dynamics [9]. These frequency domain approaches can be extended to the time domain through the use of norm bounds on the operator which relates the error on the previous trial to that on the current one – the so-called error transmission operator.

This paper begins by considering ILC schemes that take, in particular, the error on both the current and a finite number of previous trials into account. The use of errors from previous trials in the algorithm corresponds to a form of ILC (trial to trial) feedforward action and the use of current trial error knowledge is direct (or current trial) feedback action. One immediate benefit from the presence of the feedback element is that the usual advantages of feedback control, e.g., stability of the closed loop system and increased robustness, are potentially available.

The first part of the paper will demonstrate how a general class ILC schemes, where the control action is a combination of feedback action on the current trial plus feedforward action from the previous trial, can be formulated such that analysis of their stability and convergence properties is equivalent (mathematically) to well studied generic problems for a distinct sub-class of 2D linear systems known as differential and discrete linear repetitive processes. Exploiting this equivalence then leads to necessary and sufficient conditions for closed loop stability and computable bounds on the convergence rate. This, in turn, leads to fundamental limitations on achievable performance expressed in terms of basic (1D) systems theoretic properties such as the relative degree and minimum phase characteristics of the example under consideration.

In the second part of the paper, the application of norm optimal approach to a chain conveyor system is described. This is (to the best of the authors knowledge) the first ever implementation of a controller designed using 2D systems theory. Finally, some open research questions are briefly discussed - some of which are theoretically based and others which arise, in essence, from the experimental work and related studies.

2 Stability and Convergence of ILC Schemes Using Repetitive Process Theory

The state space model of the plant to be controlled by an ILC scheme is assumed to be of the following form (with $T < \infty$)

$$\begin{aligned} \dot{x}_k(t) &= Ax_k(t) + Bu_k(t),\ t \in [0, T] \\ y_k(t) &= Cx_k(t) \end{aligned} \tag{2.2}$$

where on trial k, $x_k(t)$ is the $n \times 1$ state vector, $y_k(t)$ is the $m \times 1$ output vector, and $u_k(t)$ is the $l \times 1$ vector of control inputs. If the signal to be tracked is denoted by $r_d(t)$ then $e_k(t) = r_d(t) - y_k(t)$ is the error on trial k. Also without loss of generality in this section (except where stated) we set $x_k(0) = 0,\ k \geq 1$.

The class of ILC schemes considered in this section are of the following form which, in effect, is a (static and dynamic) combination of (a finite number of) previous input vectors, the current trial error, and the errors on a finite number of previous trials. In particular, on trial $k+1$ the control input vector is calculated using

$$\begin{aligned} u_{k+1}(t) &= \sum_{i=1}^{N} \alpha_i u_{k+1-i}(t) + \sum_{i=1}^{N} K_i[e_{k+1-i}](t) \\ &+ K_0[e_{k+1}](t) \end{aligned} \tag{2.3}$$

In addition to the memory' N, the design parameters in this control law are the static scalars $\alpha_i,\ 1 \leq i \leq N$, the linear operator $K_0[\cdot](t)$ which describes the current trial error contribution, and the linear operator $K_i[\cdot](t),\ 1 \leq i \leq N$, which describes the contribution of the error on trial $k+1-i$. Next we show how the closed loop system in this case can be written as a special case of the general model of linear constant pass length repetitive processes.

First note that the open loop error dynamics can be written in convolution form as

$$e_{k+1}(t) = r_d(t) - G[u_{k+1}](t),\ 0 \leq t \leq T \tag{2.4}$$

where

$$G[u](t) = C \int_0^t e^{A(t-\tau)} B u(\tau)\, d\tau \tag{2.5}$$

Using this description, it is easily shown that the closed-loop error dynamics on trial $k+1$ can be written over $0 \le t \le T$ as

$$e_{k+1}(t) = (I + GK_0)^{-1} \left\{ \sum_{i=1}^{N} (\alpha_i I - GK_i)[e_{k+1-i}](t) + (1 - \sum_{i=1}^{N} \alpha_i) r_d(t) \right\} \tag{2.6}$$

or, equivalently, in the form

$$\hat{e}_{k+1} = L_T \hat{e}_k + b \tag{2.7}$$

where

$$\hat{e}_k(t) = \left[e_{k+1-N}^T(t), \cdots, e_k^T(t) \right]^T \tag{2.8}$$

is the so-called error super-vector.

Suppose now that $\hat{e}_k \in E_T$, where E_T is a suitably chosen Banach space, and $b \in W_T$, where W_T is a linear subspace of E_T. Then in this setting, the bounded linear operator L_T maps E_T into itself, the term $L_T \hat{e}_k$ describes the contributions of the errors on the previous N trials to the current one, and b, termed the disturbance vector, describes the contribution from external sources on the current trial. Note also that the theory which now follows applies to any ILC scheme which can be written in the abstract form (2.7).

It is now routine to argue that convergence of a closed loop ILC scheme of the form considered here as $k \to \infty$ is equivalent to stability of its linear repetitive process interpretation (i.e. (2.7)). The following is the first basic stability definition.

Definition 2.1. *An ILC scheme which can be written in the form (2.7) is said to be asymptotically stable if there exists a real scalar $\delta > 0$ such that, given any initial error $\hat{e}_0$, the sequence $\{\hat{e}_k\}_{k \ge 1}$ generated by the perturbed process*

$$\hat{e}_{k+1} = (L_T + \gamma)\hat{e}_k + b,\ k \ge 0 \tag{2.9}$$

converges strongly to a limit error $\hat{e}_\infty \in E_T$ whenever $||\gamma|| \le \delta$. , where $||\cdot||$ denotes both the norm on E_T and (later) the induced operator norm.

Note here that if $b = 0$, the limit error is $\hat{e}_\infty = 0$ and the action of the ILC scheme is to reduce any initial non-zero error exponentially to zero. Note also that this definition of stability includes a certain degree of robustness. In particular, the term γ acts as (additive) model uncertainty and, by definition, the system is required to retain asymptotic stability in the presence of small perturbations from the nominal model.

Direct application of the linear repetitive process stability theory of [10] now yields the following result.

Theorem 2.1. *An ILC scheme with closed-loop error dynamics of the form (2.6) (or, equivalently, (2.7)) is asymptotically stable and hence converges if, and only if, all roots of*

$$z^N - \alpha_1 z^{N-1} - \cdots - \alpha_{N-1} z - \alpha_N = 0 \tag{2.10}$$

have modulus strictly less than unity.

In fact, the roots of (2.10) are the spectral values of L_T in this particular case and this condition is simply the interpretation of the general result that asymptotic stability of a process described by (2.7) holds if, and only if, $r(L_T) < 1$ where $r(\cdot)$ denotes the spectral radius of its argument.

If the condition of Theorem 2.1 holds then the closed loop error dynamics converge in the norm topology of $L_p[0, T]$ to

$$e_\infty = (I + GK_{\text{eff}})^{-1} r_d \tag{2.11}$$

where the so-called effective controller K_{eff} is given by

$$K_{\text{eff}} = \frac{K}{1 - \beta} \tag{2.12}$$

where

$$\beta = \sum_{i=1}^{N} \alpha_i, \;\; K = \sum_{i=0}^{N} K_i \tag{2.13}$$

The simplest way to obtain (2.11) is to replace all variables in (2.6) by their strong limits and re-arrange.

The following result, whose proof again follows by direct application of repetitive process stability theory and is hence omitted here, gives a bound on the error sequence when Theorem 2.1 holds.

Theorem 2.2. *Suppose that the condition of Theorem 2.1 holds. Then the resulting error sequence is bounded by an expression of the form*

$$||\hat{e}_k - \hat{e}_\infty|| \leq M_1(max(||e_0||, \cdots, ||e_{N-1}||) + M_2)\lambda_e^k \tag{2.14}$$

where M_1 and M_2 are positive real scalars, and $\lambda_e \in (max|\mu_i|, 1)$ where μ_i, $1 \leq i \leq N$, is a solution of (2.10).

The result of Theorem 2.1 is counter-intuitive in the sense that stability is largely independent of the plant and the controllers used. This is a direct result of the fact that the trial duration T is finite and over such an interval a linear system can only produce a bounded output irrespective of its stability properties and in this definition of stability unstable outputs of this kind are still ‘acceptable’. Hence even if the error sequence generated is guaranteed to converge to a limit, this terminal error may be unstable and/or possibly worse than the first trial error, i.e. the use of ILC has produced no improvement in performance. To guarantee an acceptable (i.e. stable (as the most basic requirement)) limit error the stronger concept of stability along the pass (see below) has to be used.

Theorems 2.1 and 2.2 can be used to derive important results on parameter selection. In particular we have the following.

1. Convergence is predicted to be 'rapid' if λ_e is small and will be geometric in form, converging approximately with λ_e^k.

2. The limit error is nonzero but is usefully described by a (1D linear systems) unity negative feedback system with effective control K_{eff} defined by (2.12) and (2.13). If $\max_i(|\mu_i|) \to 0+$ then the limit error is essentially the first learning iterate, i.e. use of ILC has little benefit and will simply lead to the normal 'large errors' encountered in simple feedback loops. There is hence 'pressure' to let $\max_i|\mu_i|$ be close to unity when K_{eff} is a 'high gain' controller which will lead (roughly speaking) to 'small' limit errors.

3. Zero limit error can only be achieved if $b \equiv 0$ which , in turn, requires that $\sum_{i=1}^{N} \alpha_i = 1$ which is not possible if $r(L_T) < 1$ (but is possible for the case of $r(L_T) = 1$). This situation is reminiscent of classical control where the inclusion of an integrator (on the stability boundary) into the controller results in zero steady state (limit) error in response to constant reference signals.

There is a conflict in the above conclusions which has implications on the systems and control structure from both the theoretical and practical points of view. In particular, consider for ease of presentation, the case when $K_i = 0,\ 1 \leq i \leq N$. Then 'small' learning errors will require high effective gain yet GK_0 should be stable under such gains.

High gain feedback systems are described in essential detail by the system root-locus. The details are omitted here for brevity and can, for example, be found in [8]. This reference also details the limitations imposed by a non-minimum phase plant.

In what follows we consider the application of the stronger linear repetitive process stability theory concept termed stability along the pass to ILC schemes which can be written in the abstract form (2.7). The starting point is to note that there are two essential problems with asymptotic stability applied to ILC schemes.

The first of these is that only statements about the situation after an infinite number of trials have occurred are possible and little information is available concerning performance from trial to trial. The second problem is (as noted above) that although a limit error is guaranteed to exist, it could well have unacceptable dynamic characteristics. In particular, exponentially growing ('unstable') signals can be accepted because of the finite trial length and, in practice, this is clearly undesirable. In the next section, we will introduce the concept of norm optimal control to deal with the first problem and below we address the second problem using the concept of stability along the pass, termed stability along the trial here, of linear repetitive processes of the form (2.7). The key feature is that the stability along the trial property is independent of T.

To apply this to the iterative learning control schemes of this section, consider the case when the trial length (or pass length in linear repetitive process terminology) can be arbitrarily greater than the 'nominal' value T, i.e. it may become infinite. Then the formal definition of this property is as follows.

Definition 2.2. *An ILC scheme that can be written in the form (2.7) is said to be stable along the trial if there exists real numbers $M_\infty > 0$ and $\lambda_\infty \in (0, 1)$ independent of T such that, for each $T_e > T$, the error sequence from the model (2.7) satisfies the inequality*

$$||\hat{e}_k - \hat{e}_\infty|| \leq M_\infty \lambda_\infty^k \{||\hat{e}_0|| + \frac{||b||}{1 - \lambda_\infty}\} \tag{2.15}$$

Using linear repetitive process theory [10], it can be shown that the property of Definition 2.2 is equivalent to the existence of real numbers $M_\infty > 0$ and $\lambda_\infty \in (0, 1)$ independent of T such that

$$||L_T^k|| \leq M_\infty \lambda_\infty^k, \; \forall \, T_e \geq T \tag{2.16}$$

To illustrate the application of the stability along the trial property, it is instructive to consider the open loop case where the trial to trial error dynamics are defined by the integral operator $L : e_k \to e_{k+1}$

$$e_{k+1}(t) = De_k(t) + C \int_0^t e^{A(t-\tau)} B \, e_k(\tau) \, d\tau = L[e_k(.)](t), \; 0 \leq t \leq T \tag{2.17}$$

with $E_T = C([0, T]; \mathbb{R}^m)$ the space of bounded continuous real-valued functions in the interval $0 \leq t \leq T$ on which the norm is defined as $||e(t)|| = \sup_{0 \leq t \leq T} ||e(t)||_m$, where $|| \cdot ||_m$ is any convenient norm in $\mathbb{R}^m$, e.g. $||y||_m = \max_{1 \leq i \leq m} |y_i|$. In this case, asymptotic stability is easily shown to be equivalent to $r(D) < 1$ and to provide a physical explanation of this property suppose that $r(D) \leq ||D|| < 1$ (a sufficient condition) holds. Then $e_{k+1}(0) = De_k(0)$ is reduced from trial to trial, i.e. the sequence of initial errors is reduced from trial to trial (since it only depends on D). By continuity of e_{k+1}, this also occurs for t 'slightly' greater than zero. In this way, the matrix D 'squeezes' the error to zero, starting from $t = 0$ and working to $t = T$. Unfortunately, depending on the state space triple (A, B, C), it could be that for $t \gg 0$ the error actually increases over the first few trials and it takes 'a large' number of trials before the error is small everywhere. This is shown by first noting that asymptotic stability guarantees the existence of real scalars $M_T > 0$ and $\lambda_T \in (0, 1)$ such that

$$||e_k - e_\infty|| \leq M_T \lambda_T^k \{||e_0|| + \frac{||b||}{1 - \lambda_T}\}, \; k \geq 0 \tag{2.18}$$

The term involving λ_T in this last equation relates to the error reduction due to D and the term M_T relates to, and depends on, the system structure defined by the state space triple $\{A, B, C\}$. This whole process can be visualized as squeezing something out of a tube, e.g. toothpaste, where when the end is already flat, a bulge develops in the middle and after 'squeezing' long enough everything drops out at the end.

In particular, with $E_{T_e} = C([0, T_e]; \mathbb{R}^m)$ where T_e is arbitrary and not necessarily finite, we obtain the following result for stability along the trial in the case of (2.17). (The proof of this result is again omitted since it is a direct application of repetitive process stability theory.)

Theorem 2.3. *Suppose that the pair $\{C, A\}$ is observable and the pair $\{A, B\}$ is controllable. Then the ILC process (2.17) with $T_e \geq T$ is stable along the trial if, and only if,*
(a) $r(D) < 1$, $|sI_n - A| \neq 0$, $Re(s) \geq 0$; *and*
(b) *all eigenvalues of the transfer function matrix*

$$L(s) = C(sI_n - A)^{-1}B + D \tag{2.19}$$

have modulus strictly less than unity for $s = \imath\omega$, $\omega \geq 0$

3 Norm Optimal ILC

The norm optimal approach in general has a mature theoretical basis [1] and in this setting the following is the formal definition of a successful ILC algorithm.

Definition 3.1. *Consider a dynamic system with input u and output y. Let $\mathcal{Y}$ and $\mathcal{U}$ be the output and input function spaces respectively and let $r \in \mathcal{Y}$ be a desired reference trajectory from the system. Then an ILC algorithm is successful if, and only if, it constructs a sequence of control inputs $\{u_k\}_{k\geq 0}$ which, when applied to the system or plant (under identical experimental conditions), produces an output sequence $\{y_k\}_{k\geq 0}$ with the following properties of convergent learning:*

$$\lim_{k\to\infty} y_k = r, \;\; \lim_{k\to\infty} u_k = u_\infty \tag{3.20}$$

Here convergence is interpreted in terms of the topologies assumed in $\mathcal{Y}$ and $\mathcal{U}$ respectively.

Note: This general description includes linear and nonlinear dynamics, continuous or discrete plants, and time-invariant or time-varying systems.

Now let the space of output signals $\mathcal{Y}$ be a real Hilbert space and $\mathcal{U}$ also be a real (and possibly distinct) Hilbert space of input functions. Then the respective inner products (denoted by $\langle\cdot,\cdot\rangle$) and norms $||\cdot||^2 = \langle\cdot,\cdot\rangle$ are indexed in a way that reflects the space if it is appropriate to the discussion.

The dynamics of the plant considered here are approximated by a linear model which in operator form can be written as

$$y = Gu + z_0 \tag{3.21}$$

where no loss of generality arises from setting $z_0 = 0$. Also it is clear that the ILC procedure, if convergent, solves the problem $r = Gu_\infty$ for u_∞ and, if G is invertible, the formal solution is just $u_\infty = G^{-1}r$. A basic premise of the ILC approach is that the direct inversion of G is regarded as an impractical solution because it requires exact knowledge of G and involves derivatives of r. This high-frequency gain characteristic would make the approach sensitive to noise and other disturbances. Also it can be argued that inversion of the whole plant G is unnecessary as the solution only requires finding the pre-image of r under G.

The above problem is easily shown to be equivalent to finding the minimizing input u_∞ for the optimization problem

$$min_u\{||e||^2 \,:\, e = r - y, y = Gu\} \tag{3.22}$$

The optimal error $||r - Gu_\infty||^2$ is a measure of how well the ILC algorithm has solved the inversion problem. It also represents the best that the system can do in tracking the signal r. The case of interest here is when the optimal error is zero, i.e. u_∞ is a solution of $r = Gu_\infty$. Also (3.22) is clearly a singular optimal control problem which by its very nature requires an iterative solution.

In particular, the class of ILC algorithms considered here compute, at the completion of trial k, the input on trial $k+1$ as the solution of the minimum norm optimization problem

$$u_{k+1} = \arg min_{u_{k+1}}\{J_{k+1}(u_{k+1})\} \tag{3.23}$$

subject to

$$e_{k+1} = r - y_{k+1},\; y_{k+1} = Gu_{k+1} \tag{3.24}$$

where the performance index (or optimality criterion) used is

$$J_{k+1}(u_{k+1}) = ||e_{k+1}||^2_{\mathcal{Y}} + ||u_{k+1} - u_k||^2_{\mathcal{U}} \tag{3.25}$$

The initial control $u_0 \in \mathcal{U}$ can be arbitrary but, in practice, will be a good first guess at the solution of the problem. Also the relative weighting of reducing the current trial error against minimizing the deviation in the control input signals used on successive passes can be absorbed into the definitions of the norms in $\mathcal{Y}$ and $\mathcal{U}$.

The benefits of this approach are immediate from the simple interlacing result

$$||e_{k+1}||^2 \leq J_{k+1}(u_{k+1}) \leq ||e_k||^2,\; \forall\, k \geq 0 \tag{3.26}$$

which follows from optimality and the fact that the (non-optimal) choice of $u_{k+1} = u_k$ would lead to the relation $J_{k+1}(u_k) = ||e_k||^2$. This result states that the algorithm is a descent algorithm as the norm of the error is monotonically non-increasing in k and also equality holds if, and only if, $u_{k+1} = u_k$, i.e. when the algorithm has converged and no more input-updating takes place.

The controller on trial $k+1$ is given by

$$u_{k+1} = u_k + G^* e_{k+1},\; \forall k \geq 0 \tag{3.27}$$

This relationship, together with the error update relation

$$e_{k+1} = (I + GG^*)^{-1} e_k, \; \forall \, k \geq 0 \tag{3.28}$$

and the recursive input update relation

$$u_{k+1} = (I + G^*G)^{-1}(u_k + G^* r), \; \forall \, k \geq 0 \tag{3.29}$$

can be used to undertake a detailed analysis of the (theoretical) properties of this class of ILC laws [1].

In this paper, we will only consider the special case of $J_{k+1}(u_{k+1})$ defined as follows applied to a linear time invariant differential plant model with state space matrices (A, B, C) (state, input and output respectively)

$$\begin{aligned} J_{k+1}(u_{k+1}) &= \frac{1}{2}\int_0^T \{e_{k+1}^T(t) Q e_{k+1}(t) + (u_{k+1}(t) - u_k(t))^T R (u_{k+1}(t) - u_k(t))\} dt \\ &+ \frac{1}{2} e_{k+1}^T(T) F e_{k+1}(T) \end{aligned} \tag{3.30}$$

and the symmetric matrices Q, R, and F satisfy the normal linear quadratic optimal control assumptions. Standard optimal control theory now gives the solution as

$$\begin{aligned} \dot{\psi}_{k+1}(t) &= -A^T \psi_{k+1}(t) - C^T Q e_{k+1}(t) \\ u_{k+1}(t) &= u_k(t) + R^{-1} B^T \psi_{k+1}(t) \\ \psi_{k+1}(T) &= C^T F e_{k+1}(T), \; t \in [0, T] \end{aligned} \tag{3.31}$$

This representation is non-causal (in the standard sense) but it can be transformed into a causal implementation as detailed next for the case of a relaxation factor α.

Transform the costate vector $\psi_{k+1}(t)$ using

$$\psi_{k+1}(t) = -K(t)\left[x_{k+1}(t) - \alpha x_k(t)\right] + \zeta_{k+1}(t) \tag{3.32}$$

where the feedback gain matrix $K(t)$ satisfies the well known Riccati (matrix) differential equation

$$\begin{aligned} \dot{K}(t) &= -A^T K(t) - K(t) A + K(t) B R^{-1} B^T K(t) - C^T Q C \\ K(T) &= C^T F C \end{aligned} \tag{3.33}$$

Note that this last equation is independent of the inputs, states and outputs of the system and hence only needs to be computed once before the sequence of trials begin.

The predictive or 'feedforward' term $\zeta_{k+1}(t)$ must be computed on each trial using

$$\begin{aligned} \dot{\zeta}_{k+1}(t) &= -(A - BR^{-1}B^T K)^T \zeta_{k+1}(t) - \alpha C^T Q e_k(t) \\ &+ (1-\alpha) K B u_k(t) - (1-\alpha) C^T Q r(t) \end{aligned} \tag{3.34}$$

with terminal boundary condition

$$\zeta_{k+1}(T) = C^T F \left[\alpha e_k(T) + (1-\alpha) r(T)\right] \tag{3.35}$$

The algorithm is now causal since (3.34) and (3.35) can be solved off-line by reverse time simulation using available previous trial data.

Next we detail the application area studied in this paper.

4 Chain Conveyor Systems

Previous work [5] has applied three term (or PID) ILC schemes to this conveyor structure both in simulation and experiment. The major conclusion was that this is a highly relevant application area for ILC. Despite this, achievable performance was limited by the PID structure in certain cases. Hence the decision was made to apply norm optimal based ILC schemes outlined in the previous section to chain conveyor systems. The eventual goal is to assess the performance of such schemes in 'real world' operation - both stand alone and comparatively. In the remainder of this paper, we describe the chain conveyor system to be used, its mathematical modeling and configuration for the actual implementation of control action, and the design of the candidate ILC scheme.

The System

The chain conveyor systems considered in this work have two possible operational modes - indexing and synchronization. When operating in an indexing mode the conveyor moves one item at a time under a dispenser. The dispenser remains stationary and product is dispensed when the conveyor comes to rest. This motion is then repeated for the next item. In synchronization mode the conveyor moves at constant velocity and the dispenser moves back and forth. Product is dispensed when the position and velocity of the dispenser are synchronized to that of item on the conveyor. The system is measured by its accuracy combined with rate of throughput and reliability. Each requirement introduces difficulties and accuracy will degrade with time due to component wear. Commonly this is overcome by regular manual re-calibration of the system.

High rates of throughput imply large accelerations. These produce large electrical and mechanical stresses in the system components that increase wear and reduces accuracy. Ultimately high stresses will cause the premature failure of components, reducing reliability and overall throughput. It is therefore necessary to ensure that the controller demand does not require the actuator to perform outside of the manufacturers specifications. As described in [5] the system has many problems that a PID controller cannot deal with at high enough accuracy to meet typical performance requirements.

The conveyor, see Figure 1, is constructed from a 3m long framework of right angle steel section and consists of two parallel strings of 0.5 pitch steel roller chain. At 300 mm intervals there is an aluminum plate supported on a rod that is pinned through a bushing on each

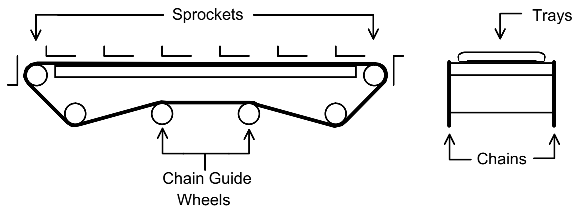

Figure 1: Schematic of Conveyor Construction.

chain. A standard squirrel cage induction motor supplied by a variable voltage variable frequency (VVVF) inverter, that is delta connected to a 3 phase pulse width modulated (PWM) inverter, drives the conveyor through a timing belt drive with a 5:2 reduction ratio. The induction motor is oversized for the mechanical load to ensure that the actuator will not limit system performance. A 500 pulse per revolution optical encoder, making measurements on the motor shaft with differential outputs, provides position feedback. Processing using a DEVA 004 motion control card increases the resolution to the equivalent of 2000 pulses per revolution.

The dispenser, see Figure 2, consists of a trolley that moves linearly above the conveyor. The trolley is an open frame, as this allows dispensing systems and instruments to be exchanged when required. A long belt supplies the linear motion, rotary motion being provided by an identical induction motor/belt drive system as for the conveyor.

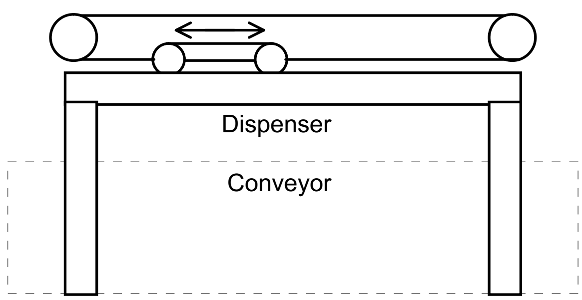

Figure 2: Schematic of Dispenser Construction.

Frequency-Domain Model

The models of the conveyor and the dispenser used are linear approximations, which were developed for simulation purposes. These were obtained by driving the conveyor and the dispenser with a variable frequency sinusoid, provided by a dc motor drive and recording the frequency response. The motor velocity was measured by using a tachometer, and then scaled to give a response relating input voltage to output velocity in counts/seconds. From the resulting Bode plots linear approximations were derived for the conveyor and dispenser respectively as

$$G_{\text{conv}}(s) \quad = \quad \frac{615.06 \times 10^6}{(s^2 + 49s + 35^2)(s^2 + 54s + 180^2)} \tag{4.36}$$

$$G_{\text{disp}}(s) \quad = \quad \frac{6.47 \times 10^6}{(s+35)(s^2+99s+110^2)} \tag{4.37}$$

Next we briefly describe how the control action is applied.

Control Implementation

A PC controls the system that includes the DEVA interface card. The card has two 14-bit Digital to Analogue Converters (D/A) for speed demand output and two optically-isolated digital outputs for axes enabling. A programmable interrupt controller is provided to produce regularly timed processor interrupts suitable for running discrete controllers. As the inverters are unable to accept a -10V to +10V speed demand, a positive analogue speed signal is provided, with a single line of the parallel port linked to the direction setting pin of the inverter to provide direction control. A program, written in C, provides a user interface to the hardware and also implements the controller.

System Simulation

In order to begin the evaluation of the performance of norm optimal ILC designs in this area, including the relative advantages/disadvantages of predictive action, a simulation of the system operating in synchronous mode has been constructed in MATLAB/SIMULINK. Also a range of controllers have been designed. For example, Figure 3 shows a sample design (for the dispenser transfer function) where the reference signal used is the same as that used in the previous work on the use of PID ILC for this application [5].

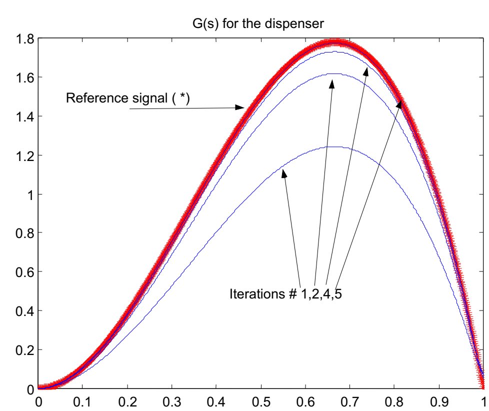

Figure 3: Illustrative Design.

5 Conclusions

The goal of the research programme on which this paper is based is to evaluate the performance (both stand alone and comparative terms) of norm optimal based ILC schemes in the ‘real world’ operating domain. The testbed chosen for this is a chain conveyor system. To date, the experimental testbed has been constructed and the relevant parts of its dynamics approximated by linear models, in the form of transfer functions constructed from measured frequency domain data, obtained. Also a range of controllers based on both the norm optimal and predictive norm optimal ILC designs have been completed (a sample norm optimal design has been included here).

This paper has described the necessary background development to undertake an extensive range of experimental tests which will be used to address the following key questions (and others).

1. How do normal optimal and predictive norm optimal ILC compare against alternatives (from, in the main, [5] and the relevant cited references).

2. Are there any benefits to be obtained by using predictive norm optimal ILC [2], i.e. an enhancement of norm optimal control action where the (weighted) effects of the errors on a finite number of future trials are taken into account in the cost function to be minimized, against just norm optimal ILC.

3. If norm optimal ILC does indeed give improved (relative) performance, how can this be quantified in terms of the key extra parameters in these control laws, i.e. the prediction horizon N and the weighting factor λ?

4. What are the general messages from this study in terms of the theme of this special session, i.e. the relative merits of higher order ILC.

Further early results from the experimental programme (beyond those of Figure 3 here) can be found in [4]. Initial assessment of them does indeed confirm the promise of the norm optimal approach to iterative learning controller design for what is a major industrial applications area (chain conveyor systems are extensively used in many process control areas, such as bread production systems). The following areas require short to medium term further research.

1. The effects of unmodeled dynamics.

2. The effects of mismatch in resetting the process initial conditions.

3. The development of the algorithms in a stochastic setting to deal with the effects of noise.

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