MODELING OF OUT-OF-PLANE HYGROINSTABILITY OF MULTI-PLY PAPERBOARD

GIANANTONIO BORTOLIN, PER OLOF GUTMAN, BENGT NILSSON

Abstract. This report describes a semi-physical model for the dimensional stability properties (i.e. curl) of the carton board produced at AssiDomän Frövi, Sweden. The main equations are based on classical lamination theory of composite materials, and each constituent ply is considered as a macroscopic homogeneous, elastic medium. The model used data from February to June 2001, and show a general agreement between predicted and measured curvatures. The data required are those related to the elastic, and expansional properties of the board, and those considered relevant to determine the moisture content in each layer of the cartonboard.

1. Introduction

Curl in paper is an important problem since the departure from the flat may seriously affect the processing of the paper. For this reason, customers impose quite restrictive limits on the allowed curvatures of the board. So, it is becoming more and more important to be able to produce a cartonboard with a curl within certain limits. Due to the economic significance of the curl problem, much research has been performed to find sheet design and processing strategies to eliminate or reduce curl.

At AssiDomän Frövi (Sweden), the problem of dimensional instability causes the loss of several tons of board every year, and it is one of the most complicated quality variable to handle for the operators and process engineers. At present, curl is measured once every hour from samples taken at the end of each tambour. After conditioning for a few minutes, the curl and twist measurements are taken by an optical device with a good accuracy. However, the curl may vary considerably across the web, and a previous investigation showed that the standard deviation of the measurements is very high if compared with their mean value. In this scenario, it is clear that a curl predictor/simulator would be a very useful tool for the operators and process engineers in order to help them to decide the best settings and/or control action to take.

This project takes its inspiration from a previous work of Jens Petterson [19] on a bending stiffness predictor, which is nowadays used at the paper plant.

This report is structured in the following way: first a short overview on dimensional stability and related properties is presented. Afterwards, the process and the model is explained, and finally the simulation results are shown with a few comments on the results.

Key words and phrases. Grey box modeling, curl.

2. Dimensional stability: some background

Curl in paper and paperboard is defined as departure from flat form. Three curl components (machine direction, cross-machine direction, and diagonal curl or twist) characterize the magnitude of curl. The three curvatures at moderate rotations may be expressed as:

$$K_x = -\frac{\partial^2 w}{\partial x^2}, \quad K_y = -\frac{\partial^2 w}{\partial y^2}, \quad K_{xy} = -2\frac{\partial^2 w}{\partial x \partial y} \tag{2.1}$$

where $w(x, y)$ is the out-of-plane displacement of the sheet $[m^{-1}]$, x is the Machine Direction (MD), and y the Cross Direction (CD). The out-of-plane deviation of the sheet is then approximate as follows:

$$w(x, y) = -\frac{1}{2}K_x x^2 - \frac{1}{2}K_y y^2 - \frac{1}{2}K_{xy} xy \tag{2.2}$$

In most cases, curl is a manifestation of dimensional instability, reflecting a difference in this property through the thickness of the paper. The structure of the paper is therefore directly involved in the extension and direction of curl. The primary cause for curl is then the intra-fiber shrinkage and expansion with changes in Relative Humidity (RH), and the communication of this dimensional instability to the web.

3. Modelling

As we stated in the introduction, the main purpose of the model is to have a tool for better understanding the process, and also for model predictive control. The modelling approach we use is based on grey-box modelling (see [2], [19]). The reasons for such an approach is that the physical process is very complex and nonlinear. The influence of some inputs is not entirely understood, and besides, it depends on a number of unknown parameters and unmodeled/unmesurable disturbances.

The board is composed of four layers, with two identical middle ones. Hence, as a first approximation we consider the board as three-layered. There are six different pulp qualities, some of which are produced at Frövi, from various raw materials (birch and pine), others are purchased like the CTMP (Chemo Thermo Mechanical Pulp). Some of the pulp qualities are refined, including the reject pulp produced from rejected board. It is well known in the literature that beating increases flexibility in the fibers and so also their hygroexpansivity. However, in our case the refiners are controlled in such a way that the different pulps have more or less a constant water retention value (WRV). In this way, also the refining energies are more or less constant during the normal production. Since it was not possible to make experiments due to high costs, the refining energies were not identifiable and they were not included in the model.

Then, the pulp flows to the 4 headboxes which spread it into the wire sections where the first drying takes place. The distribution of the fibers in the wire is dependent on many factors such as the suspension acceleration in the slice channel of the headbox, the speed difference between the suspension and the wire, and turbulence on the wire (see [16] for more details). Typically, in paper-making fiber orientation is controlled by means of the jet-to-wire speed difference. In KM5 the speed difference is measured on-line for all the layers, so it was used together with the tensile ratio to model the fiber orientation dependence of the hygroexpansivity coefficients β.

After the wires, the four layers are pressed together and then the board is fed to the drying section. According to the literature (see e.g. [20]), the early stage of the drying has little influence on the hygroexpansivity properties of the paper, so only the last group of cylinders is taken into consideration in the model. In fact, the steam pressures of the last cylinders group is also used by the operators to control the curl in MD direction.

After the drying section, steam is added to the top and to the bottom of the board which is then pressed together by two hot calenders.

Then, the top layer is coated by two coating devices. After each section, the top layer is dried by infrared dryers, and by hot hoods. After the coating sections, the bottom layer is wetted by a device called LAS. The amount of water added to the bottom layer is decided by the operators, and it is used to control CD curl. The bottom layer is then dried by infrared dryers and hot hoods. Finally, the board passes through the last drying group, and another calender section.

The proposed model is a Multi-Input-Multi-Output, non-linear, static model. The inputs are the following:

1: Layers thickness $[m]$.
2: Layers densities $[Kg/m^3]$.
3: The tensile stiffness indexes of the layers $[Nm/Kg]$.
4: Pulp fractions [%].
5: Jet to wire speed ratio of top and bottom layer.
6: Steam pressures of the last drying cylinders (Group 6-7)$[Pa]$.
7: Steam added to the top layer before calendering $[Kg/m^2]$.
8: Pressures of all three calendering sections [Pa].
9: Temperatures of the calendering sections [Celsius].
10: Total coating $[Kg/m^2]$.
11: Total water added to the bottom layer by LAS $[Kg/m^2]$.
12: Speed of the machine at the wire and at the pope $[m/min.]$

The final model can be divided into different parts, as shown in Fig. 1. The main one is the mechanical model, which is described in detail in Appendix A. It is based on a paper by Carlsson (see [1]), where the classical laminate theory is used to model the dimensional stability of multi-ply board. Despite its simplicity, the model agreed satisfactory with the data for moderate rotations, and low moisture content (between 40 and 50 % RH). The main assumption in Carlsson's analysis, is that each ply is considered as an homogeneous elastic medium. The mechanical model takes as inputs the strains of the three layers, their thickness, the densities and their elastic moduli. The latter ones are calculated using Petterson's bending stiffness predictor [19]. The outputs of the model are the curvatures $[m^{-1}]$ in the MD, CD, and diagonal directions.

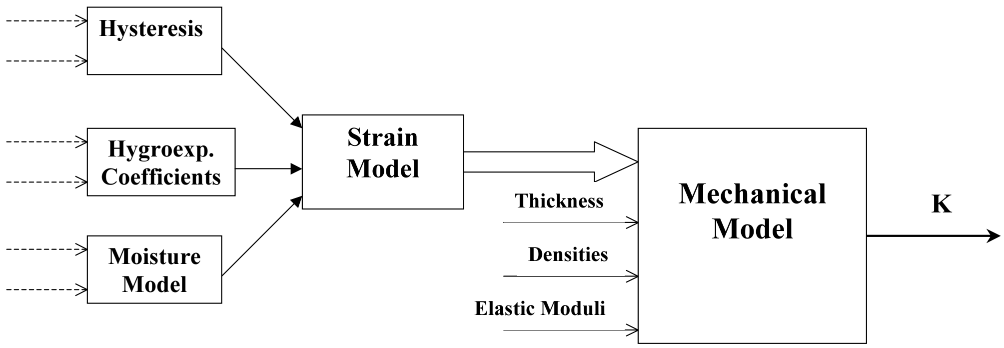

FIGURE 1. Structure of the model.

The strain was modelled by the following equation:

$$\epsilon_{ij} = \epsilon_{ij}^0 + \beta_{ij} H_i \qquad \text{[dimensionless]} \tag{3.3}$$

where the indices i and j relate to the layer (i.e. top, middle, bottom) and the direction (i.e. MD, CD, diagonal), β is the hygroexpansivity coefficient calculated by eq. (3.7), H_i is the moisture of layer i, and ϵ_{ij}^0 is a bias term that takes into account the effect of internal stresses, s_{ij}^0, developed during the paper-making process. These internal stresses, s_{ij}^0, were modelled in the following way:

$$s_{ij}^0 = s_{ij0} + a_{ij} e^{b_i} \qquad [\text{N/m}^2] \tag{3.4}$$

where the indices i and j relate to the layer and the direction, s_{ij}^0 and a_{ij} are parameters to be identified. The variable b_i is a linear combination of the control inputs that contribute to the drying stresses in layer i, causing the irreversible shrinkage, such as the water added (i.e. coating, LAS, etc.), the drying temperatures, the draw, and the three calenders pressures and temperatures. These internal stresses, s^0, generate a force and a momentum that in last analysis will produce the irreversible shrinkage (i.e. ϵ^0). In appendix A, a more detailed description of these stress-strain relationships is given.

The moisture model is based on the assumption that the the final moisture is a property of the paper composition, since the curl is measured in a controlled environment after that the paper has been conditioned in an oven for about 20 minutes.

The hygroexpansivity coefficients were modelled using the following simplified linear approximation (see [16], [15], and [14]):

$$\beta_{ij} = \beta_{ij0} + \theta_{ij}(1) R_i + \theta_{ij}(2) S_i + \theta_{ij}(3) \rho_i \tag{3.5}$$

where the indices i and j relate to the layer, and to the curl direction, β_{ij0} is the hygroexpansivity potential defined below, R_i is the ratio between the tensile stiffness index in MD and CD directions, S_i is the jet to wire speed ratio, ρ_i is the density, and θ is a vector of parameters to be identified.

We introduce here the concept of hygroexpansivity potentials q_{jk} associated with each kind of pulp k. These new quantities are equivalent to the density potential in [19], and they will be estimated from data. Given these pulp potentials, the hygroexpansivity potentials of each ply are calculated simply by:

$$\beta_{ij0} = \sum_k f_{ik} q_{jk} \tag{3.6}$$

where f_{ik} is the weight fraction of pulp k in layer i.

3.1. **Parameter Identification.** The resulting model has 78 parameters which can be estimated using classical predictor error methods (see [11]). Using Ljung's notation, the model can be written as an output error model:

$$y(t) = G(q, u(t), \theta) + e(t) \tag{3.7}$$

where $y(t)$ is the vector of curvatures, $u(t)$ is the vector of inputs, $G(u(t), \theta)$ is the nonlinear function describing the model, θ is the vector of parameters, and $e(t)$ is the error, which is assumed to be white noise with covariance matrix Λ_0. Note that in this case, the one-step-ahead prediction at time l, $\hat{y}_j(l)$, is equal to the one-step-ahead simulation. We define the residuals, or predictor errors as:

$$\varepsilon_j(l) = y_j(l) - \hat{y}_j(l) \tag{3.8}$$

where the index j relates to the direction (CD, MD, diagonal), $y_j(l)$ is the measured curvature at time l, and $\hat{y}_j(l)$ is the predicted value. The estimation of the parameters was done by first introducing the following loss function (see [19]):

$$V_N(\theta, Z^N) = \frac{1}{N} \sum_{l=1}^{N} [\varepsilon_{CD}^2(l)/\lambda_{CD} + \varepsilon_{MD}^2(l)/\lambda_{MD} + \varepsilon_{TW}^2(l)/\lambda_{TW}] \tag{3.9}$$

where N is the number of samples, Z^N is the input-output data, and λ_j are weights used to normalize the square predictor errors. Then, the estimated parameter vector, $\hat{\theta}_N$, is defined as the value of θ that minimizes (3.9):

$$\hat{\theta}_N = \arg\min_{\theta} V_N(\theta, Z^N) \tag{3.10}$$

In order to evaluate the "quality" of the estimated parameters their standard deviations was also calculated. Given the estimated vector $\hat{\theta}_N$ which minimizes the loss function $V_N(\theta, Z^N)$ (eq. 3.9), the covariance matrix is given by:

$$\mathrm{Cov}(\hat{\theta}_N) \simeq \frac{1}{N} P_\theta \tag{3.11}$$

where P_θ for a finite number of data can be approximated by:

$$\hat{P}_N = \Big[\frac{1}{N} \sum_{k=1}^{N} \frac{\partial \varepsilon(k)}{\partial \theta} \Lambda^{-1} \frac{\partial \varepsilon(k)^T}{\partial \theta}\Big]^{-1} \tag{3.12}$$

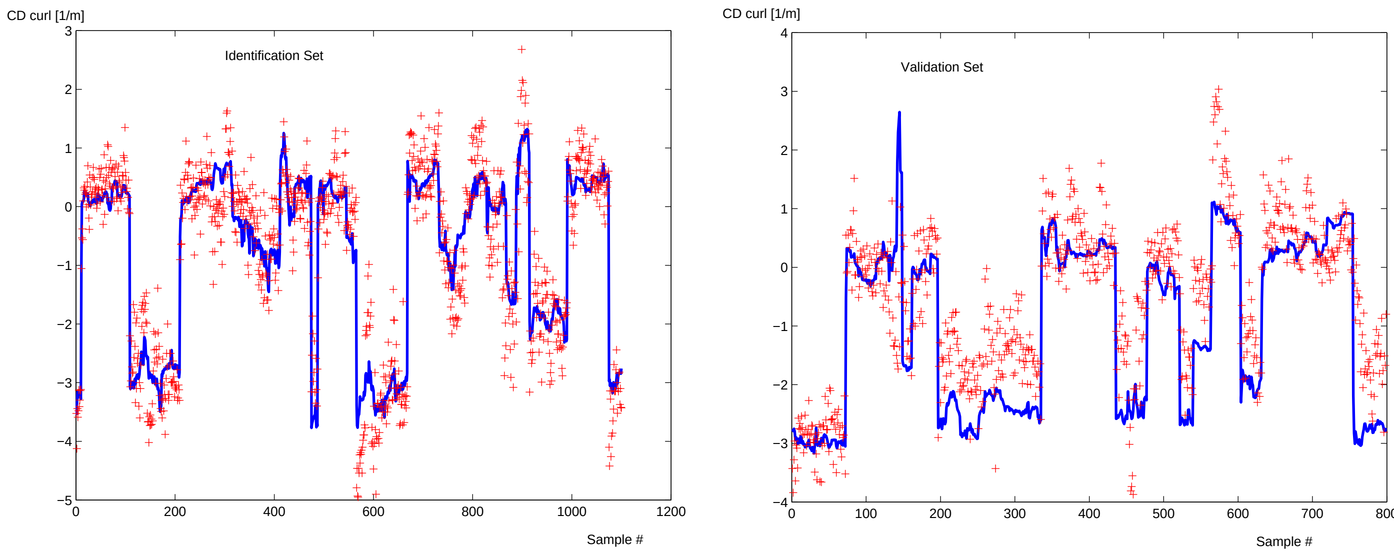

FIGURE 2. Simulated (solid line) and measured ('+') CD curl the identification data (on the left), and for the validation data (on the right).

where $\Lambda = \text{diag}(\lambda_j) = \Lambda_0$. If $\Lambda \neq \Lambda_0$, the expression for $\hat{P}_N$ becomes more complicate (see Ljung [11]).

For the estimation, we used about 1100 samples collected during the production periods from October to December 2001. The minimization of the function (3.9) was made using the command *lsqnonlin* from the Optimization Toolbox in Matlab. This function uses a trust region method algorithm, and it is described in [18]. The loss function (3.9) is nonlinear in the parameters, and it is not convex. So, it is not guaranteed that the function *lsqnonlin* finds the global minimum. However, the minimization was repeated for different initial conditions, and also an elliptical random search algorithm was employed to verify if the obtained value of the parameters vector was the optimal.

In 3.12, the term between square parenthesis is an approximation of the Hessian matrix of V_N. Successful minimization is only possible when the Hessian is well conditioned, that is when the ratio of the highest to the lowest eigenvalues is not too high. When it is, only a subset of the parameter vector θ can reliably be estimated. Unfortunately, in our case the Hessian is ill conditioned. In this case, some of the parameters have to be fixed to their nominal value, and the previous optimization routine can be used to estimate the remaining parameters. In our case we decided to fix to their initial values all the parameters that have a standard deviations larger than their value, and the remaining 64 parameters were identified again.

This ill-conditioned problem is caused by the fact that the identification set is not rich enough (i.e. informative enough) for the identification. In fact, the paper machine is always driven with more or less the same settings, and it was not possible to make any experiments due to the high costs.

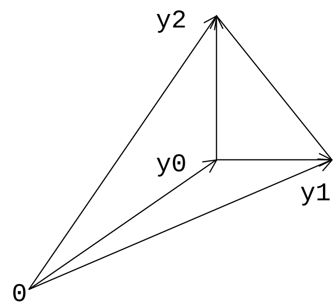

Figure 3. Approximate correlation diagram in the curl-space. y_0 is the true value of the curvature, y_1 is the measured value, and y_2 is the predicted value.

4. Results and Discussion

For the sake of brevity, only the simulated CD curvatures are shown in Fig. 2, for both the identification and validation data set. For the validation we used data collected in a period between January to February 2002. The results of the simulations show that there is a general agreement between the model and the measurement. In fact, the model is most of the time able to capture the "average" value of all three the curvatures. However, the predictions are quite bad in some period of time for both the identification and validation sets. A possible reason may be the fact that the model does not include all the significant input variables that influence the dimensional stability of the cartonboard. Another possible reason may be that the linear mechanical model (Appendix A) is not accurate enough.

Besides, the validation simulations show a bias in the predicted curvature, which is also increasing with time. We suppose that this bias is due to the time variations of the parameters.

In order to evaluate the model more carefully, the standard deviations of the residuals were computed. Ideally, they should be as close as possible to the standard deviations of the measurements. An elegant geometrical interpretation of the *a posteriori* standard deviation of the various predictor errors can be analyzed in Fig. 3 (see [8]). Let us denote by y_0 the "true" value of the curvature, y_1 the measured value, and by y_2 the predicted value. Assuming that the "true" predictor error $y_0(k)-y_2(k)$, the measurement error $y_0(k)-y_1(k)$, and the true curvature are mutually uncorrelated. Then:

$$\sigma^2(y_2(k) - y_0(k)) = \sigma^2(\varepsilon(k)) - \sigma^2(y_1(k) - y_0(k)) \tag{4.13}$$

where ε is the prediction error. By using (4.13) we can calculate the standard deviation of the true error. The results are shown in Tab. 1 for both the identification and the validation sets.

The "true" standard deviations are quite larger than the measurements standard deviations, even in the case of the identification set of data. This confirms the previous short analysis based on the plots about the un-reliability of the model for some periods.

We conclude, then, that the model is not accurate enough yet to be used for predictive model control. However, it may still be used by the operators as a decision-support tool, since it is able to estimate the effect of each input on the curl. For instance, an higher or

m^{-1}	Id data: Std of resid.	Val. data: Std of resid.	Id data: Std of "true" error	Val. data: Std of "true" error	Std of meas.
CD Curl	0.559	0.822	0.494	0.779	0.26
MD Curl	0.232	0.327	0.176	0.290	0.15
Twist	0.469	0.574	0.408	0.525	0.23

Table 1. Standard deviations of the measurements, of the residuals, and of the "true" error for the identification and validation data set.

lower value of the jet to wire speed ratio may give different results in different operating conditions, and the model may be used for testing in advance such control actions. Besides, it can be also used to get a better physical insight into the process. For example, we can see which are the variables with larger influence on dimensional stability and also how they effect curl.

In conclusion, the model has to be improved in order to be used in an effective way by the operators. Even though a general agreement between the model and the measurements is achieved, the standard deviation of the residuals is too high for practical operations. For the future, we are planning to make a more careful study of process, especially for the periods where the model seems to fail, in order to understand if there are important control variables neglected in the previous analysis. We are also planning to consider a more sophisticate mechanical model which includes nonlinear kinematics (see [17]). In fact, as Nordström points out in his paper ([17]), the deflections of curled paper may amount to several times the board thickness, and so effects of large deformation should be included in the analysis. Besides, in order to take into consideration the effect of the large disturbances influencing the board manufacturing (i.e. unmodelled inputs, inputs uncertainties, and modelling errors), we also intent to add a model for the disturbances, by using a nonlinear Kalman filter (see [19]).

Appendix A. A few concepts on lamination theory of composite materials

A.1. **Main assumptions.** Let's indicate the plane coordinates with j, l, and s(=shear component). We will also use the notation 1,2,6 to indicate the plane coordinates.

Since paper has non-uniform structures in the thickness direction, such as variations of mass density, fiber orientation and fiber composition, in this report we will consider paper as a composite laminate which consists of an arbitrary number of laminae with different anisotropic, mechanical, hygrotermal properties. Since our major interest is curl, the mechanical and hygrotermal properties are assumed to be independent of the spatial coordinates j and l.

A.2. **Application to paperboard.** We will assume here that the reader is familiar with the basics notions of laminate theory (see [22]). Under the initial assumptions, and using the thin-plate approximation of the classical lamination theory, the following equation can be used:

$$\sigma_l = \sum_{j=1}^{3} Q_{l,j}(\varepsilon_j - e_j) \tag{A.14}$$

where ε is the strain component, σ $[N/m^2]$ is the stress component [1], e is the expansional strain, and Q is the plane stress reduced matrix. To simplify the notation we will write the sums (Einstein's summation) in the following way:

$$\sigma_i = Q_{lj}(\varepsilon_j - e_j) \quad (= \sum_{j=1}^{3} Q_{l,j}(\varepsilon_j - e_j))$$

where the sum in j is omitted.

The variation of the strain ε through the thickness, z, is, in accordance with the classical lamination theory, described by the following equation:

$$\varepsilon_j = \varepsilon_j^0 + zK_j \qquad (j = 1, 2, 6) \tag{A.15}$$

where ε_j^0 and K_j are mid-plane strains and curvatures respectively.

The expansional strains are assumed to be linear functions of the moisture content over the humidity range of interest:

$$e_l = \beta_l H \qquad (l = 1, 2, 6) \tag{A.16}$$

where β_l is the coefficient of hygroexpansivity (or swelling coefficient) and H is the moisture content [2].

The resultant force, $N(t)$, and moment, $M(t)$, per unit of width of the laminate are given by:

$$N(t) = \int_{z_0}^{z_N} \sigma(z,t)dz, \qquad M(t) = \int_{z_0}^{z_N} z\sigma(z,t)dz \tag{A.17}$$

As we stated before, in the following analysis we suppose that the stress vector, σ, is time independent, and constant in each ply. If we combine the previous equations, we get the constitutive relation for each ply. So, we get:

$$N_l = N_l^0 + \sum_{k=1}^{N} \int_{z_{k-1}}^{z} (Q'_{l,j})_k(\varepsilon_j^0 + zK_j - (\beta_j)_k H_k)dz \qquad (l, j = 1, 2, 6) \tag{A.18}$$

$$M_l = M_l^0 + \sum_{k=1}^{N} \int_{z_{k-1}}^{z} (Q'_{l,j})_k(\varepsilon_j^0 + zK_j - (\beta_j)_k H_k)zdz \qquad (l, j = 1, 2, 6) \tag{A.19}$$

[1] Stress $[N/m^2]$ is a measure of internal forces within a body. Strain is defined as the spatial variation of a laminate displacements

[2] H is defined as the amount of moisture divided by the mass of the dry paper, so β and H have no dimension

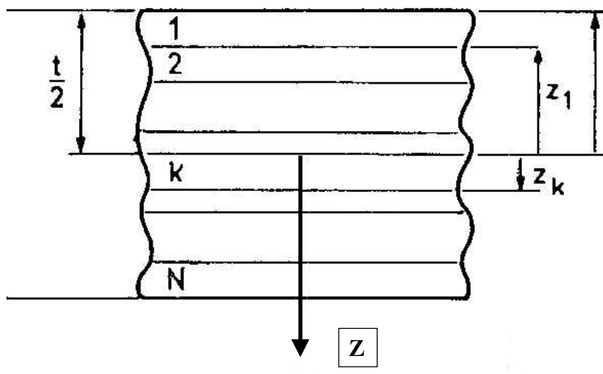

FIGURE 4. Definition of the z coordinates, z_k.

where N_l^0, and M_l^0 are the force and the moment built up by the internal stresses, s_j^0, during the papermaking process described in section 4.4 and in eq. 40-41, and the ply coordinates, z_k [m], are defined in Fig. 1. Since investigations of curl are in practice performed when the paperboard is free to deform, no external forces N_l or moments M_l are acting. So from the previous equation, by imposing $N_l = M_l = 0$ we can derive the curvatures K_l.

Let's define the following matrices:

$$A_{lj} = \sum_{k=1}^{N} (Q'_{l,j})_k (z_k - z_{k-1}) \qquad (l, j = 1, 2, 6) \tag{A.20}$$

$$B_{lj} = \frac{1}{2} \sum_{k=1}^{N} (Q'_{l,j})_k (z_k^2 - z_{k-1}^2) \qquad (l, j = 1, 2, 6) \tag{A.21}$$

$$D_{lj} = \frac{1}{3} \sum_{k=1}^{N} (Q'_{l,j})_k (z_k^3 - z_{k-1}^3) \qquad (l, j = 1, 2, 6) \tag{A.22}$$

$$N_l^0 = \sum_{k=1}^{N} (Q'_{l,j})_k (s_j^0)_k (z_k - z_{k-1}) \qquad (l, j = 1, 2, 6) \tag{A.23}$$

$$M_l^0 = \sum_{k=1}^{N} (Q'_{l,j})_k (s_j^0)_k (z_k^2 - z_{k-1}^2) \qquad (l, j = 1, 2, 6) \tag{A.24}$$

$$F_l = \sum_{k=1}^{N} H_k (Q'_{l,j})_k (\beta_j)_k (z_k - z_{k-1}) - N_l^0 \qquad (l, j = 1, 2, 6) \tag{A.25}$$

$$G_l = \frac{1}{2} \sum_{k=1}^{N} H_k (Q'_{l,j})_k (\beta_j)_k (z_k^2 - z_{k-1}^2) - M_l^0 \qquad (l, j = 1, 2, 6) \tag{A.26}$$

Then, the curvatures K_l can be calculated by the following equation:

$$K_l = [B^{*-1}](G_j - [A^*]F_j) \qquad (l, j = 1, 2, 6) \tag{A.27}$$

α	Fiber orientation angle [rad]	MD	Machine direction
β	Hygroexpansivity coefficient [-]	N	Number of samples
ϵ	Strain [-]	q	Pulp hygroexpansivity potential [-]
ε	Predictor error	Q	Plane stress matrix [N/m^2]
θ	Parameter vector	R	MD-CD tensile stiffness ratio
Λ_0	Error covariance matrix	TW	Diagonal direction
ρ	Density [Kg/m^3]	V_N	Loss function
σ	Stress [N/m^2]	w	Basis weight [Kg/m^2]
CD	Cross-machine direction	i	i-th layer
f	Weight fraction [%]	j, l	Curl direction
H	Moisture [Kg water/Kg dry paper]	k	Pulp kind
K	Curvature [m^{-1}]	t	Top layer
κ	Pulp moisture potential	m	Middle layer
S	Jet to wire speed ratio	b	Bottom layer

TABLE 2. Symbols and indexes used in the paper

where:

$$A^* = BA^{-1}$$

and

$$B^* = -BA^{-1}B + D$$

References

[1] Carlsson, L. (1981) *Out-of-Plane Hygroinstability of Multi-Ply Paperboard*, Fiber Sci. Tech. 14:201-212

[2] Bohlin T. (1996). *Interactive System Identification: Prospects and Pitfalls*, Springer Verlag, 1991

[3] Chen A. C., Borch J., Garcia C. E., et al. Journal of Imaging Sci., 29(1):11 (1985)

[4] Edwards P. J., Murray A. F., Papadopulus G., Wallace A. R., Barnard J., Smith G. (1999) *Paper Curl Prediction and Control Using Neural Networks*, TAPPI Journal

[5] Eriksson L.E., Cavlin S., Fellers C., Carlsson L. (1987) *Curl and Twist of paperboard-Theory and Measurement*, Nordic Pulp and Paper Research Journal No. 2

[6] Gallay, W. (1973). *Stability of Dimensions and Form of Paper*, Tappi Journal vol. 56, No.12

[7] Gutman, P.O. (2001) *Investigation of standard deviation when measuring curl and twist in the carton laboratory.* Frövi Internal Report.

[8] Gutman P. O., Nilsson B. (1996). *Modelling and Predictor of Bending Stiffness for Paper Board Manufactoring.* IFAC'96

[9] Ioslovich I., Seginer I., Gutman P.O. (2002) *Estimation of crop model parameters with the Nicolet lettuce Model as an example.* In preparation.

[10] Kartovaara I.,Rajala R., Luukkala M., et al. (1985). *Papermaking Raw Materials.* Mech. Eng. Publ., London, vol. 1, pp. 381-412.

[11] Ljung, L. (1987) *System Identification-Theory for the user*, Prentice-Hall, Eglewood Cliffs, New Jersey, 1987.

[12] Ljung, L. (2000) *System Identification Toolbox-User's Guide* The MathWorks.

[13] Nanri, Y. Uesaka, T. (1993) *Dimensional Stability of Mechanical Pulps-Drying Shrinkage and Hygroexpansivity*,Tappi Journal Vol. 76, No. 6

[14] Niskanen, K.J. (1998) *Paper Physics* Papermaking Science and Technology, Vol. 16

[15] Niskanen, K.J. (1996) *Curl Variations in paper and Board*, Paper and Timber Vol. 78, No. 5

[16] Niskanen, K.J. (1989) *Distribution of Fiber Orientation in Paper*, in Fundamentals of Papermaking, Baker C. F. and Punton V. W. editors, Mech. Eng. Publ. Ltd. london

[17] Nordström A., Gudmundson P., Carlsson L. (1998) *Influence of Sheet Dimensions on Curl of Paper*, Journal of Pulp and Paper Science Vol. 24, No. 1

[18] *Optimization Toolbox - User 's Guide* The MathWorks Inc., (2000).

[19] Petterson J. (1998). *On Model Based Estimation of Quality Variables for Paper Manufacturing.* Licentiate Thesis, Department of Automatic Control, KTH Stockholm, Sweden.

[20] Salmen L., Fellers C., Htun M. (1987) *The Development and Release of Dried-in Stresses in paper*, Nordic pulp and Paper Research Journal, No.2

[21] Salmen L., Boman R., Fellers C. (1987) *The Implications of Fiber and Sheet Structure for the Hygroexpansivity of Paper* Nordic Pulp and Paper Research Journal No. 4

[22] Tsai S.W., Hahn H. T. (1980) *Introduction to Composite Materials*, Technomic Publishing Company

[23] Uesaka, T. (1991) *Dimensional Stability of Paper: Upgrading Paper Performance in End Use*, Journal of Pulp and Paper Science Vol 17, No. 2

[24] Uesaka, T. (1994) *General Formula for Hygroexpansion of Paper*, Journal of Materials Science Vol. 29 pag. 2373-2377

[25] Uesaka, T., Kodaka I., Okushima S., Fukuchi R. (1989) *History-Dependent Dimensional Stability of Paper*, Rheological Acta Vol. 28, No. 3

[26] Åström, K. J. (2001) *Control problems in paper making: revisited*, Pulp and Paper Canada Vol. 102, No. 6

Gianantonio Bortolin, Optimization and Systems Theory, Dep. of Mathematics, Royal Inst. of Technology, 100 44 Stockholm, Sweden

E-mail address: `bortolin@math.kth.se`

Per Olof Gutman, Optimization and Systems Theory, Dep. of Mathematics, Royal Inst. of Technology, 100 44 Stockholm, Sweden

E-mail address: `peo@math.kth.se`

Bengt Nilsson, AssiDomän Frövi AB, 71040 Frövi, Sweden. Tel: +46 581 37241

E-mail address: `Bengt.Nilsson@asdo.se`