Successive Stablization of a Class of 2D Systems

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Abstract

In this paper we report further significant progress on the development of a 'mature' systems theory for discrete linear repetitive processes which are a distinct class of 2D discrete linear systems of both systems theoretic and applications interest. Here we first propose an extension to the basic state space model of these processes to include coupling terms previously neglected but which could arise in applications. Then we develop some significant first results on the analysis and control of examples represented by this new model structure.

1 Introduction and Background

The essential unique characteristic of a repetitive, or multipass, process is a series of sweeps, termed passes, through a set of dynamics defined over a fixed finite duration known as the pass length. On each pass an output, termed the pass profile, is produced which acts as a forcing function on, and hence contributes to, the next pass profile. This, in turn, leads to the unique control problem for these processes in that the output sequence of pass profiles generated can contain oscillations that increase in amplitude in the pass to pass direction.

To introduce a formal definition, let $\alpha < +\infty$ denote the pass length (assumed constant). Then in a repetitive process the pass profile $y_k(t)$, $0 \le t \le \alpha$, (t being the independent temporal or spatial variable) generated on pass k acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile $y_{k+1}(t)$, $0 \le t \le \alpha$, $k \ge 0$.

Physical examples of repetitive processes include long-wall coal cutting and metal rolling operations [2, 11]. Also a number of so-called algorithmic examples exist where adopting a repetitive process setting for systems related analysis has clear advantages over alternative approaches. These include classes of iterative learning control schemes [1] and iterative solution algorithms for nonlinear dynamic optimal control problems based on the maximum principle [8].

Attempts to control these processes using standard (or 1D) systems theory/algorithms fail (except in a few very restrictive special cases) precisely because such an approach ignores their inherent 2D systems structure, i.e. information propagation occurs from pass to pass and along a given pass, and the effects of resetting the initial conditions before the start of each new pass. In seeking a rigorous foundation on which to develop a control theory for these processes, it is natural to attempt to exploit structural links which exist between, in particular, the class of so-called discrete linear repetitive processes and 2D linear systems described by the extensively studied Roesser [6] or Fornasini Marchesini [3] state space models. Discrete linear repetitive processes are distinct from such 2D linear systems in the sense that information propagation in one of the two separate directions (along the pass) only occurs over a finite duration. This is also the distinction between differential (the dynamics along each pass evolves as a function of a continuous variable) linear repetitive processes and the 2D continuous-discrete linear systems studied by Kaczorek [7] and others.

In common with a large range of other areas in systems theory, recent years has seen the emergence of Linear Matrix Inequality (LMI) based techniques in the analysis of differential and discrete linear repetitive processes respectively (see, for example, [5] in the discrete case). This has led to considerable success, especially in areas such as the following which have proved difficult to advance using other analysis tools.

1. Computable conditions for stability (the underlying stability theory for these processes can be found in [9] and is based on an abstract model in a Banach space setting which includes all processes with linear dynamics and a constant pass length as special cases) in the presence physically relevant types of uncertainty in the matrices of the defining state space model.

2. The design of practically relevant control algorithms/laws.

It is important to note, however, that this progress has often required the preliminary step of constructing the 1D discrete linear systems equivalent state space model description of the underlying process dynamics and the structure of this model also results in the need to solve (potentially) very large dimensioned LMIs. Also the model structure considered to date neglects components (reflecting the rich and widely varying dynamics which repetitive processes can exhibit) which could be very important in some applications areas. In this paper, we first propose a model structure to capture these missing components and then develop significant new results on the control related analysis of processes described by this new model. These results consist of those which are the (non-trivial) extension of ones already in existence for other models of discrete linear repetitive processes (e.g., the property of so-called pass controllability [4]) and others on so-called successive stabilization which are completely new.

This paper considers the class of so-called extended discrete linear repetitive processes whose state space model has the following structure over $0 \leq p \leq \alpha - 1,\ k \geq 0$,

$$\begin{aligned} x_{k+1}(p+1) &= Ax_{k+1}(p) + Bu_{k+1}(p) + \sum_{j=0}^{\alpha-1} B_j y_k(j) \\ y_{k+1}(p) &= Cx_{k+1}(p) + Du_{k+1}(p) + \sum_{j=0}^{\alpha-1} D_j y_k(j) \end{aligned} \tag{1.1}$$

Here on pass k, $x_k(p)$ is the $n \times 1$ state vector, $y_k(p)$ is the $m \times 1$ pass profile vector, and $u_k(p)$ is the $r \times 1$ vector of control inputs.

In order to complete the process description, it is necessary to specify the initial conditions, i.e. the pass state initial vector sequence and the initial pass profile, which are also termed the boundary conditions in the repetitive process literature. This is a critical task since it is known (see [10] in the case of differential linear repetitive processes) that the structure of these initial, or boundary, conditions alone can cause instability for the discrete linear repetitive process state space models considered to date. Here, however, these are assumed to be of the simplest (but still practically relevant) form

$$\begin{aligned} x_{k+1}(0) &= d_{k+1},\ k \geq 0 \\ y_0(p) &= y(p),\ 0 \leq p \leq \alpha - 1 \end{aligned} \tag{1.2}$$

where d_{k+1} is an $n \times 1$ vector with known constant entries and $y(p)$ is an $m \times 1$ vector whose entries are known functions of p over $0 \leq p \leq \alpha - 1$.

Suppose now that $\forall p = 0, 1, \ldots, \alpha - 1$

$$B_j = \begin{cases} B_0 & j = p \\ 0 & j \neq p \end{cases} \tag{1.3}$$

and also

$$D_j = \begin{cases} D_0 & j = p \\ 0 & j \neq p \end{cases} \tag{1.4}$$

Then the model of (1.1) is equivalent to the linear repetitive process state space model first introduced in [9].

Motivation for considering processes of the form (1.1) arises from applications where the current pass profile at any point along the pass is a function of more than one point on the previous pass. In the discrete linear repetitive process state space model first proposed in [9], it was assumed that the current pass and state profile vector was only directly influenced by the pass profile vector at the same point on the previous pass. Here we study the case when all points along the previous pass directly influence the state and pass profile vectors at any point on the current pass and, in particular, processes with state space model (1.1) which is one possible way of representing this feature.

2 Results

In this paper, we will report on progress so far in following areas relating to the systems theoretic/controller design related analysis of processes described by (1.1) and (1.2).

- Extension of the abstract model based stability theory [9] and the subsequent development of computationally feasible stability tests.

- The construction of the 1D discrete linear systems equivalent state space model description of the underlying dynamics.
- The use of this 1D equivalent model in stability analysis and the characterization of so-called pass controllability. (As with other classes of 2D linear systems, there is more than one distinct concept of controllability for discrete linear repetitive processes.)
- Successive stabilization. This is a new method of designing control laws (based on current pass state feedback action augmented by feedforward action from the previous pass) in such a way that only small (in relative terms) LMIs need to be solved.

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