Adaptive Predictive Control with Controllers of Restricted Structure

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Abstract

The application of novel adaptive predictive optimal controllers of low order, that involve a multi-step cost index and future set-point knowledge, is considered. The usual predictive controller is of high order and the aim is to utilise simpler structures, for applications where PID controllers might be employed for example. A non-linear system is assumed to be represented by multiple linear discrete-time state-space models, where n of these models are linearisations of the underlying non-linear system at an operating point, determined off-line. One extra model is identified on-line. The optimisation is then performed across this range of $N_f + 1$ models to produce a single low order control law. One advantage of this approach is that it is very straightforward to generate a much lower order predictive controller and thereby simplify implementation. Also, with respect to the adaptive nature of the algorithm, the solution is rather cautious. Each new update of the controller involves averaging the cost function across both fixed and currently identified models, providing robust adaptive control action. The method is applied to a piecewise non-linear system, implemented by switching between several linear systems, and results are given.

1 Introduction

Predictive optimal control is used extensively in industry for applications such as large-scale supervisory systems [1]. Predictive control depends upon the assumption that future reference or setpoint information is available, which may then be incorporated into the optimal control law to provide improved tracking characteristics and smaller actuator changes.

The best known predictive control approach is probably Dynamic Matrix Control (DMC), which was introduced for complex multivariable plants with strong interactions and competing constraints [2]. DMC has been applied in more than 1000 plants worldwide and aims to drive a plant to the lowest operating cost. The algorithm includes a steady state optimiser based on the economics of the process so that set points can be manipulated to optimise the total system. The focus of this type of commercial algorithm is at the supervisory levels of the control hierarchy where the order of the controller is not such a problem. If predictive control is to become widely adopted at the regulating level there is a need for low-order simple controller structures, and this is the problem addressed.

The predictive control algorithms based upon multi-step cost-functions and the receding horizon control law, were generalized by Clarke and coworkers in the Generalized Predictive Control (GPC) algorithm [3]. Future set-point information has been used in a number of Linear Quadratic (LQ) optimal control problems and summarized in the seminal work of Bitmead et al [4]. The use of state-space models for Generalized Predictive Control (GPC) was proposed in [5] and extended in [6].

Multi-step cost-functions may also be used in LQ cost-minimization problems. The solution of the multi-step Linear Quadratic Gaussian Predictive Control (LQGPC) problem, when future set point information is available, has been considered in [7], when the plant is represented in polynomial matrix form. The solution of the LQGPC cost minimization problem for systems represented in state equation form was given in [8] and [9]. There are a number of model predictive control philosophies which employ state equation models which are related to these results, such as in [10]. The solution strategy followed is to minimise an H2 or LQG criterion in such a way that the predictive controller is of the desired form and is causal. A simple analytic solution cannot be obtained, as in the case where the controller structure is unconstrained [11]. However, a relatively straightforward direct optimization problem can be established which provides the desired solution.

The aim of this paper is to present a new method of generating adaptive low-order predictive optimal controllers that could be used in non-linear control applications. This simplification is to be achieved without losing the benefits of either the multi-step criterion or the future set-point knowledge.

2 System Model

The system shown in Fig.1 is represented by the linear, time-invariant, discrete-time state-space system representation given below, where the state vector $X(t) = \begin{bmatrix} x_0(t) & x_1(t) \end{bmatrix}^T$ is a combination of the states for both reference generator and plant:

$$X(t+1) = \begin{bmatrix} A_0 & 0 \\ 0 & A_1 \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ B_1 \end{bmatrix} u(t) + \begin{bmatrix} D_0 & 0 \\ 0 & D_1 \end{bmatrix} \begin{bmatrix} \xi_0(t) \\ \xi_1(t) \end{bmatrix} \tag{2.1}$$

$$X(t+1) = AX(t) + Bu(t) + D\xi(t) \tag{2.2}$$

$$z_1(t) = \begin{bmatrix} 0 & 0 \\ 0 & C_1 \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ v_1(t) \end{bmatrix} \tag{2.3}$$

$$z_1(t) = CX(t) + v_1(t), y_h(t) = H_1 x_1(t), r_h(t+p) = H_r x_{r0}(t) \tag{2.4}$$

The states $x_1(t) \in \mathbb{R}^n$, $x_0(t) \in \mathbb{R}^p$, control input $u(t) \in \mathbb{R}$, white noise disturbance $\xi_1(t) \in \mathbb{R}$, observation $z_1(t) \in \mathbb{R}$, white output noise $v_1(t) \in \mathbb{R}$, inferred output $y_h \in \mathbb{R}$, driving white noise input $\xi_0(t) \in \mathbb{R}$, and inferred reference $r_h(t) \in \mathbb{R}$.

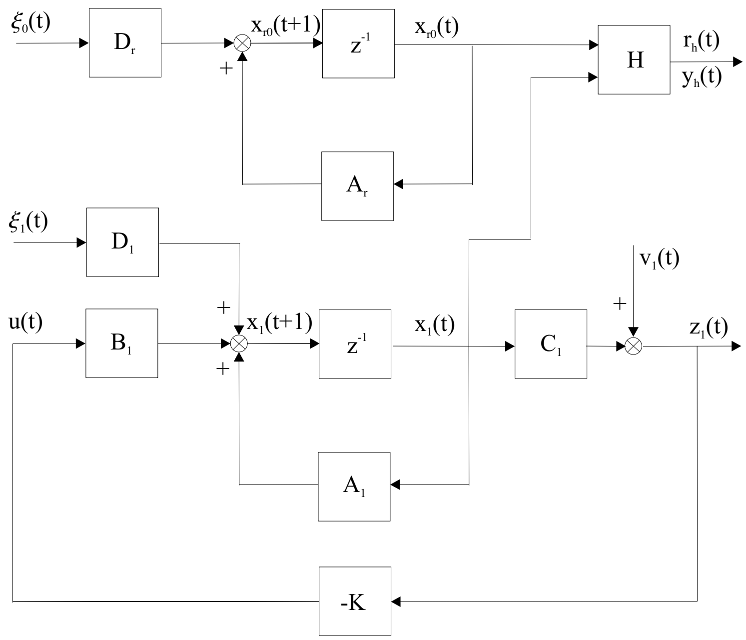

Figure 1: Plant Model and Reference Generator

To produce the reference signals at {t+1,t+2,...,t+p-1}, the $x_0(t)$ state is created by delaying $x_{r0}(t)$. Hence:

$$x_0(t) = \begin{bmatrix} x_{r0}(t) \\ x_{r1}(t) \\ x_{r2}(t) \\ \vdots \\ x_{r(p-1)}(t) \end{bmatrix}, A_0 = \begin{bmatrix} A_r & 0 & \cdots & & 0 \\ H_r & 0 & \cdots & & \vdots \\ 0 & 1 & & & \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}, D_0 = \begin{bmatrix} D_r \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{2.5}$$

To produce the vector of future reference values from the state vector, the following matrix-vector product is formed:

$$\begin{bmatrix} r_h(t+1) \\ r_h(t+2) \\ \vdots \\ r_h(t+p-1) \\ r_h(t+p) \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 1 & 0 \\ \vdots & \vdots & & & \vdots \\ 0 & 1 & \cdots & 0 & 0 \\ H_r & 0 & \cdots & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{r0}(t) \\ x_{r1}(t) \\ \vdots \\ x_{r(p-2)}(t) \\ x_{r(p-1)}(t) \end{bmatrix} \tag{2.6}$$

$$R_{t+1,N} = H_0 x_0(t) \tag{2.7}$$

with an obvious definition of terms in (2.7).

Also, the matrices C and D are partitioned to give $C_{11} = \begin{bmatrix} 0 & 0 \end{bmatrix}$, $C_{21} = \begin{bmatrix} 0 & C_1 \end{bmatrix}$, $D_{11} = \begin{bmatrix} D_0 \\ 0 \end{bmatrix}$, and $D_{12} = \begin{bmatrix} 0 \\ D_1 \end{bmatrix}$. These partitions are used later, in the definition of the system transfer function matrices.

Having established the plant equations, an estimator is required to predict the inferred output for j steps ahead. The estimator is stated below:

$$Y^h_{t+1,N} = H_N x_1(t) + G_N U_{t,N} + N_N W_{t,N} \tag{2.8}$$

$$Y^h_{t+1,N} = \begin{bmatrix} y_h(t+1) \\ y_h(t+2) \\ \vdots \\ y_h(t+N) \end{bmatrix}, \; H_N = \begin{bmatrix} H_1 A_1 \\ H_1 A_1^2 \\ \vdots \\ H_1 A_1^N \end{bmatrix}$$

$$G_N = \begin{bmatrix} H_1 B_1 & 0 & \cdots & 0 \\ H_1 A_1 B_1 & H_1 B_1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ H_1 A_1^{N-1} B_1 & H_1 A_1^{N-2} B_1 & \cdots & H_1 B_1 \end{bmatrix}, U_{t,N} = \begin{bmatrix} u(t) \\ u(t+1) \\ \vdots \\ u(t+N-1) \end{bmatrix}$$

$$N_N = \begin{bmatrix} H_1 D_1 & 0 & \cdots & 0 \\ H_1 A_1 D_1 & H_1 D_1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ H_1 A_1^{N-1} D_1 & H_1 A_1^{N-2} D_1 & \cdots & H_1 D_1 \end{bmatrix}, W_{t,N} = \begin{bmatrix} \xi_1(t) \\ \xi_1(t+1) \\ \vdots \\ \xi_1(t+N-1) \end{bmatrix}$$

3 Predictive control problem formulation

For a scalar system with white noise input signals, the predictive control performance index to be minimised can be defined in the time domain as in [12]:

$$\begin{aligned} J &= E\left\{ \lim_{T\to\infty} \frac{1}{2T} \sum_{t=-T}^{T} J_t \right\} \\ J_t &= \sum_{j=1}^{N} Q_j (r_h(t+j) - y_h(t+j))^2 + \sum_{j=0}^{N-1} R_j u(t+j)^2 \end{aligned} \tag{3.1}$$

where $E\{.\}$ is the unconditional expectation operator and y_h and r_h are the inferred output and reference signals respectively. The error and control weightings, Q_j and R_j, need not remain fixed over the sequence of j's.

By expressing the system description in the state-space form of Section 2 it is possible, with suitable manipulation, to restate the cost function in frequency domain form [13]:

$$\begin{aligned} J_p &= E\left\{\lim_{T\to\infty}\frac{1}{2T}\sum_{t=-T}^{T} X^T(t)\bar{Q}_c X(t) + u^T(t)\bar{R}_c u(t) + 2X^T(t)\bar{G}_c u(t)\right\} \\ &= \frac{1}{2\pi j}\oint_{|z|=1} trace\{\bar{Q}_c\Phi_{XX}(z^{-1}) + 2\bar{G}_c\Phi_{uX}(z^{-1}) + \bar{R}_c\Phi_{uu}(z^{-1})\}\frac{dz}{z} \end{aligned} \tag{3.2}$$

where Φ_{XX}, Φ_{uu}, and Φ_{uX} are the power spectrums of the state, control input and the cross-spectrum of state and control input respectively. To obtain the $\bar{Q}_c$, $\bar{R}_c$, and $\bar{G}_c$ matrices it is necessary to first partition $\tilde{G} = -G_N$, and $\tilde{R} = diag\{R_0, \ldots R_{N-1}\}$ to match the partitioning of $U_{t,N}$ into current and future controls:

$$\tilde{G} = \begin{bmatrix} G_{N1} & G_{N2} \end{bmatrix}, \tilde{R} = \begin{bmatrix} R_0 & 0 \\ 0 & \tilde{R}_{22} \end{bmatrix}$$

where $\tilde{R}_{22} = diag\{R_1, \ldots, R_{N-1}\}$. Noting that $\tilde{Q} = diag\{Q_1, \ldots Q_N\}$, $\tilde{H} = \begin{bmatrix} H_0 & -H_N \end{bmatrix}$, $Q_c = \tilde{H}^T\tilde{Q}\tilde{H}$, $R_c = \tilde{G}^T\tilde{Q}\tilde{G} + \tilde{R}$, and $G_c = \tilde{H}^T\tilde{Q}\tilde{G}$, the definitions of R_c and G_c can now be expressed in terms of these partititions:

$$R_c = \begin{bmatrix} R_{c1} & R_{c3} \\ R_{c3}^T & R_{c2} \end{bmatrix} = \begin{bmatrix} G_{N1}^T\tilde{Q}G_{N1} + R_0 & G_{N1}^T\tilde{Q}G_{N2} \\ G_{N2}^T\tilde{Q}G_{N1} & G_{N2}^T\tilde{Q}G_{N2} + \tilde{R}_{22} \end{bmatrix} \tag{3.3}$$

$$G_c = \begin{bmatrix} G_{c1} & G_{c2} \end{bmatrix} = \begin{bmatrix} -\tilde{H}^T\tilde{Q}G_{N1} & -\tilde{H}^T\tilde{Q}G_{N2} \end{bmatrix} \tag{3.4}$$

The desired matrices are then defined as: $\bar{Q}_c = Q_c - G_{c2}R_{c2}G_{c2}^T$, $\bar{R}_c = R_{c1} - R_{c3}R_{c2}^{-1}R_{c3}^T$ and $\bar{G}_c = G_{c1} - G_{c2}R_{c2}^{-1}R_{c3}^T$.

3.1 Polynomial H_2 problem solution

In order to produce the optimal control law for the given system, the spectral factors and Diophantine equations below must first be solved:

Spectral Factors

$$D_{cp}^*D_{cp} = \bar{B}_{1p}^*\bar{Q}_c\bar{B}_{1p} + \bar{A}_{1p}^*\bar{R}_c\bar{A}_{1p} + \bar{B}_{1p}^*\bar{G}_c\bar{A}_{1p} + \bar{A}_{1p}^*\bar{G}_c^*\bar{B}_{1p} \tag{3.5}$$

$$D_{dp}D_{dp}^* = C_{dp}C_{dp}^* + A_{dp}R_{f1}A_{dp}^* \tag{3.6}$$

Diophantine Equations

$$z^{-g_1} D^*_{cp} G^{c*}_{1p} + F^c_{1p} \bar{A}_p = (\bar{B}^*_{1p} \bar{Q}_c + \bar{A}^*_{1p} \bar{G}^*_c) z^{-g_1} \tag{3.7}$$

$$z^{-g_1} D^*_{cp} H^{c*}_{1p} - F^c_{1p} \bar{B}_p = (\bar{A}^*_{1p} \bar{R}_c + \bar{B}^*_{1p} \bar{G}_c) z^{-g_1} \tag{3.8}$$

$$z^{-g_2} G^f_{1p} D^*_{dp} + \bar{A}_p F^f_{1p} = D_{12} C^*_{dp} z^{-g_2} \tag{3.9}$$

$$z^{-g_2} H^f_{1p} D^*_{dp} - C_{21} z^{-1} F^f_{1p} = R_{f1} A^*_p z^{-g_2} \tag{3.10}$$

The various polynomial matrices are obtained from the system transfer functions defined below:

Resolvent Matrix: $\Phi(z^{-1}) = (zI - A)^{-1}$

Plant models: $\bar{W}(z^{-1}) = \Phi(z^{-1})B$, $W(z^{-1}) = C_{21}\Phi(z^{-1})B$

Disturbance Models: $\bar{W}_d(z^{-1}) = \Phi(z^{-1})D_{12}$, $W_d(z^{-1}) = C_{21}\Phi(z^{-1})D_{12}$

Reference Models: $\bar{W}_r(z^{-1}) = \Phi(z^{-1})D_{11}$, $W_d(z^{-1}) = C_{11}\Phi(z^{-1})D_{11}$

Letting $\bar{A}_p = (I - z^{-1}A)$, the right coprime form of $\bar{W}$ may be written as:

$$\bar{W} = \bar{A}_p^{-1} \bar{B}_p = \bar{B}_{1p} \bar{A}_{1p}^{-1} \tag{3.11}$$

where $\bar{B}_p = z^{-1}B$. Also, the left-coprime forms for W and W_d may be written as:

$$W = A_p^{-1} B_p, W_d = A_{dp}^{-1} C_{dp} \tag{3.12}$$

Ultimately, the optimal control problem reduces to minimising

$$J_d^+ = \frac{1}{2\pi j} \oint_{|z|=1} (T_d^+ T_d^{+*}) \frac{dz}{z} \tag{3.13}$$

where

$$\begin{aligned} T_d^+ \;=\; & H^c_{1p}([1 + H^{c-1}_{1p} G^c_{1p} \bar{A}_p^{-1} (I + G^f_{1p} H^{f-1}_{1p} A_p^{-1} \bar{C}_{2p})^{-1} \bar{B}_p] K - H^{c-1}_{1p} G^c_{1p} \bar{A}_p^{-1} z^{-1} \\ & (I + G^f_{1p} H^{f-1}_{1p} A_p^{-1} \bar{C}_{2p})^{-1} G^f_{1p} H^{f-1}_{1p})(A_p + B_p K)^{-1} D_{dp} \end{aligned} \tag{3.14}$$

This is achieved when $T_d^+ = 0$. The optimal feedback control law, K, is therefore:

$$K = K_c [z\bar{A}_p + K_{f1} C_{21} + BK_c]^{-1} K_{f1} \tag{3.15}$$

where $K_c = H^{c-1}_{1p} G^c_{1p}$ and $K_{f1} = G^f_{1p} H^{f-1}_{1p}$.

4 Restricted Structure and Adaptive Control

4.1 Restricted Structure solution

The optimal solution to the predictive optimal control problem simply requires T_d^+ to be set to zero. In the case of a restricted structure control law, it is necessary that (3.13) be minimised with respect to the parameters of the given controller structure. In the following analysis, it will be assumed that K is a modified-PD controller:

$$K_{modPD} = K_p + K_d \frac{(1 - z^{-1})}{1 - \tau z^{-1}} \tag{4.1}$$

Therefore, the controller parameters of interest in this case are K_p and K_d.

Making the appropriate substitutions in (3.14), as in (3.15), obtain:

$$T_d^+ = H_{1p}^c([1 + K_c S_f B]K_{modPD} - K_c S_f K_{f1})(A_p + B_p K_{modPD})^{-1} D_{dp} \tag{4.2}$$

where $S_f = (z\bar{A}_p + K_{f1}C_{21})^{-1}$.

Rewriting K_{modPD} as a rational function,

$$\begin{aligned} K = \frac{K_n}{K_d} &= \frac{K_p(1 - \tau z^{-1}) + K_d(1 - z^{-1})}{(1 - \tau z^{-1})} \\ &= \frac{K_p \alpha_0 + K_d \alpha_1}{\alpha_0} \end{aligned} \tag{4.3}$$

where α_0 and α_1 have the obvious definitions, T_d^+ becomes:

$$T_d^+ = K_n L_{n1} - K_d L_{n2} \tag{4.4}$$

where

$$L_{n1} = L_1/(K_n L_3 + K_d L_4) \ \ and \ \ L_{n2} = L_2/(K_n L_3 + K_d L_4) \tag{4.5}$$

$$L_1 = H_{1p}^c[1 + K_c S_f B]D_{dp} \ , \ L_2 = H_{1p}^c K_c S_f K_{f1} D_{dp} \ , \ L_3 = B_p \ , \ L_4 = A_p \tag{4.6}$$

T_d^+ is obviously non-linear in K_p and K_d, rendering (3.13) particularly difficult to minimise directly. However, an iterative solution is possible if the values of K_p and K_d in the denominator of T_d^+ are assumed known.

Defining $L_1 = H_{1p}^c[1 + K_c S_f B]D_{dp}$, $L_2 = H_{1p}^c K_c S_f K_{f1} D_{dp}$, $L_3 = B_p$, and $L_3 = A_p$

$$T_d^+ = \frac{L_1 K_n - L_2 K_d}{L_3 K_n + L_4 K_d} \tag{4.7}$$

Defining $L_{n1} = L_1/(L_3 K_n + L_4 K_d)$, $L_{n2} = L_2 K_d/(L_3 K_n + L_4 K_d)$, T_d^+ becomes linear in K_n:

$$T_d^+ = L_{n1} K_n - L_{n2} \tag{4.8}$$

As T_d^+ is a complex function and the complex conjugate is required in (3.13), the next step is evidently to split the elements of T_d^+ into real and imaginary parts, denoted by the superscripts ^{r} and ^{i}:

$$\begin{aligned} T_d^+ &= (L_{n1}^r + jL_{n1}^i)(K_n^r + jK_n^i) - (L_{n2}^r + jL_{n2}^i) \\ &= L_{n1}^r K_n^r - L_{n1}^i K_n^i - L_{n2}^r + j(L_{n1}^i K_n^r + L_{n1}^r K_n^i - L_{n2}^i) \end{aligned} \tag{4.9}$$

Splitting K_n

$$K_n^r = K_p \alpha_0^r + K_d \alpha_1^r, K_n^i = K_p \alpha_0^i + K_d \alpha_1^i \tag{4.10}$$

and substituting:

$$\begin{aligned} T_d^+ &= K_p((L_{n1}^r \alpha_0^r - L_{n1}^i \alpha_0^i) + j(L_{n1}^i \alpha_0^r + L_{n1}^r \alpha_0^i)) \\ &\quad + K_d((L_{n1}^r \alpha_1^r - L_{n1}^i \alpha_1^i) + j(L_{n1}^i \alpha_1^r + L_{n1}^r \alpha_1^i)) - (L_{n2}^r + jL_{n2}^i) \end{aligned} \tag{4.11}$$

Noting that $T_d^+ T_d^{+*} = |T_d^+|^2 = (T_d^{+r})^2 + (T_d^{+i})^2$, it is obvious that the integrand of (3.13) can be represented by a matrix-vector product:

$$T_d^+ T_d^{+*} = \begin{bmatrix} T_d^{+r} & T_d^{+i} \end{bmatrix} \begin{bmatrix} T_d^{+r} \\ T_d^{+i} \end{bmatrix} = (Fx - L)^T (Fx - L) \tag{4.12}$$

where

$$F = \begin{bmatrix} (L_{n1}^r \alpha_0^r - L_{n1}^i \alpha_0^i) & (L_{n1}^r \alpha_1^r - L_{n1}^i \alpha_1^i) \\ (L_{n1}^i \alpha_0^r + L_{n1}^r \alpha_0^i) & (L_{n1}^i \alpha_1^r + L_{n1}^r \alpha_1^i) \end{bmatrix} \tag{4.13}$$

and

$$L = \begin{bmatrix} L_{n2}^r \\ L_{n2}^i \end{bmatrix}, x = \begin{bmatrix} K_p \\ K_d \end{bmatrix} \tag{4.14}$$

The complex integral cost is evaluated for $|z| = 1$. Hence, the matrices can be expressed as a function of the real frequency variable, ω:

$$\begin{aligned} J_d^+ &= \frac{1}{2\pi j} \oint_{|z|=1} (T_d^+(z^{-1}) T_d^{+*}(z^{-1})) \frac{dz}{z} \\ &= \frac{T}{2\pi} \int_0^{2\pi/T} (F(e^{-j\omega T})x - L(e^{-j\omega T}))^T (F(e^{-j\omega T})x - L(e^{-j\omega T})) d\omega \end{aligned} \tag{4.15}$$

The cost function can be optimised directly, but a simple algorithm is obtained if the integral is approximated by a summation with a sufficient number of frequency points, $\{\omega_1, \ldots, \omega_k, \ldots, \omega_N\}$. That is:

$$\begin{aligned} J_d^+ &\approx \sum_{k=1}^{N} (F(e^{-j\omega_k T})x - L(e^{-j\omega_k T}))^T (F(e^{-j\omega_k T})x - L(e^{-j\omega_k T})) \\ &= (b - Ax)^T (b - Ax) \end{aligned} \tag{4.16}$$

where

$$A = \begin{bmatrix} F(e^{-j\omega_1 T}) \\ \vdots \\ F(e^{-j\omega_N T}) \end{bmatrix}, b = \begin{bmatrix} L(e^{-j\omega_1 T}) \\ \vdots \\ L(e^{-j\omega_N T}) \end{bmatrix}, x = \begin{bmatrix} K_p \\ K_d \end{bmatrix} \tag{4.17}$$

Assuming that the matrix $A^T A$ is non-singular, the least squares optimal solution is:

$$x = (A^T A)^{-1} A^T b \tag{4.18}$$

Of course, as the assumption was made that the solution x was already known in the denominator of T_d^+, this is a case where the method of successive approximation, as in [14], can be used. This involves a transformation T such that $x_{n+1} = T(x_n)$. Under appropriate conditions, the sequence $\{x_n\}$ converges to a solution of the original equation. Since this optimisation problem is non-linear there may be not be a unique minimum. However, the algorithm presented in the next subsection does always appear to converge to an optimal solution in many industrial examples.

4.2 Adaptive control

The adaptive controller to be described is based on the multiple-model version of a restricted-structure optimal controller. This version is so called due to the use of a set of mathematical models to represent a single non-linear or time-varying system at different operating points. The aim is to produce a single controller which will stabilise the entire set of models. The cost function employed is a weighted sum of costs for individual system representations.

Let J_{dj}^+ denote the value of (4.16) for the jth system model, and let the probability of this model being the true representation be denoted by p_j. Also, let the b and A matrices in (4.16) for the jth system model be b_j and A_j respectively. Then the multiple-model cost criterion can be written as:

$$
\begin{aligned}
\bar{J}_d^+ &= \sum_{j=1}^{n+1} p_j J_{dj}^+ \\
&= \sum_{j=1}^{n+1} p_j (b_j - A_j x)^T (b_j - A_j x) \\
&= (\underline{b} - \underline{A}x)^T \underline{P} (\underline{b} - \underline{A}x)
\end{aligned} \tag{4.19}
$$

where

$$
\underline{A} = \begin{bmatrix} A_1 \\ \vdots \\ A_{n+1} \end{bmatrix}, \underline{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_{n+1} \end{bmatrix}, \underline{P} = diag\{p_1, \ldots, p_{n+1}\} \tag{4.20}
$$

The solution to this problem is obviously similar to the single-model case. Assuming that $\underline{A}^T \underline{P} \underline{A}$ is non-singular, the least squares optimal solution is:

$$
x = (\underline{A}^T \underline{A})^{-1} \underline{A}^T \underline{b} \tag{4.21}
$$

A controller for a non-linear system can then be produced by defining the first N_f linear models to represent the non-linear system at different operating points. The adaptation is introduced by continually updating model $N_f + 1$ with recursively identified parameters and recalculating the values for x online. The following successive approximation algorithm as in Luenberger [14], with a system identification algorithm incorporated, can be used to compute the restricted structure LQG adaptive controller.

Algorithm 4.1 (Adaptive restricted-structure control algorithm). .

1. *Define N (number of frequency points), $\omega_1, \ldots, \omega_N$, τ and N_f (number of fixed models)*
2. *Initialise $K_p = K_d = 1$ (arbitrary choice)*
3. *Define $\alpha_0(z^{-1})$, $\alpha_1(z^{-1})$ (using (4.3))*
4. *Compute $C_{0n}(z^{-1}) = K_p \alpha_0(z^{-1}) + K_d \alpha_1(z^{-1})$*
5. *Compute $C_{0d}(z^{-1}) = \alpha_0(z^{-1})$*
6. *For $j = 1$ to N_f*
   (a) *Solve for the spectral factors D_{cpj} and D_{dpj}, and the Diophantine equations for G_{1pj}^c, H_{1pj}^c, F_{1pj}^c and G_{1pj}^f, H_{1pj}^f, F_{1pj}^f.*
   (b) *Create L_{1j}, L_{2j}, L_{3j}, L_{4j}, L_{n1j}, and L_{n2j}.*

(c) *For all chosen frequencies, calculate* $F_j(e^{-j\omega T})$, $L_j(e^{-j\omega T})$.

(d) *Assemble* $A_j = \begin{bmatrix} F_j(e^{-j\omega_1 T}) \\ \vdots \\ F_j(e^{-j\omega_N T}) \end{bmatrix}$ *and* $b_j = \begin{bmatrix} L_j(e^{-j\omega_1 T}) \\ \vdots \\ L_j(e^{-j\omega_N T}) \end{bmatrix}$

7. *Estimate current* A_p, B_p, *and* C_{dp} *polynomials using a recursive least squares algorithm.*

8. *Repeat steps 6(a) to (d) for the identified polynomials.*

9. *Stack the* $N_f + 1$ A *and* b *matrices to form* $\underline{A}$ *and* $\underline{b}$

10. *Calculate the restricted-structure controller gains,* $x = (\underline{A}^T P \underline{A})^{-1} \underline{A}^T P \underline{b}$

11. *If the cost is lower than the previous cost, repeat steps 8 to 10 using the new* C_{0n}. *Otherwise, use previous controller gains to compute the feedback controller* $C_{0n}(z^{-1}) = K_p\alpha_0(z^{-1}) + K_d\alpha_1(z^{-1})$ *and* $C_0(z^{-1}) = C_{0n}(z^{-1})/C_{0d}(z^{-1})$.

12. *Implement controller in feedback loop and go back to step 7.*

5 Adaptive Control of Switched Linear Models

A ship roll control problem will now be considered. Ship roll control systems are often used on passenger ferries in order to maintain a comfortable ride for passengers. The ship can be modelled by a second order transfer function, where the input is fin angle and the output is ship roll angle [15]. The natural frequency of the transfer function changes over time, dependent upon the sea state. In this case, the $N_f = 4$ fixed models that we have for the ship are for damping ratio, $\zeta = 0.5$ and natural frequency, ω_n of 0.1, 0.125, 0.15, and 0.175 rad/s. Details are given in the Appendix. The disturbance is white noise passed through an integrator.

The results presented in this section, in Figures 2 to 4, are for a 200 second simulation where the ship is represented by a second order transfer function with damping ratio of 0.5 which increases in natural frequency. ω_n begins at 0.1rad/s and increases by 0.025rad/s every forty seconds until reaching 0.2rad/s. In this way, each of the fixed model representations is covered plus an extra unknown model at 0.2rad/s. Probability of 0.2 is given to each of the fixed models plus the model identified by recursive least squares. The error prediction horizon in (3.1) is 2 steps and the control horizon is 1 step. The weights are $Q_1 = 100$, $Q_2 = 10$, $R_0 = 10^{-2}$ and $R_1 = 10^{-3}$. The adaptive control scheme is expected to identify the model parameters and tune a PD controller to give an optimal solution across the set.

Figure 2 depicts the step-reference following capability of the system. This is somewhat unrealistic, as in practice the desired ship roll angle is zero degrees. However, a square wave input shows the closed-loop response more clearly. The overshoot and settling time of the

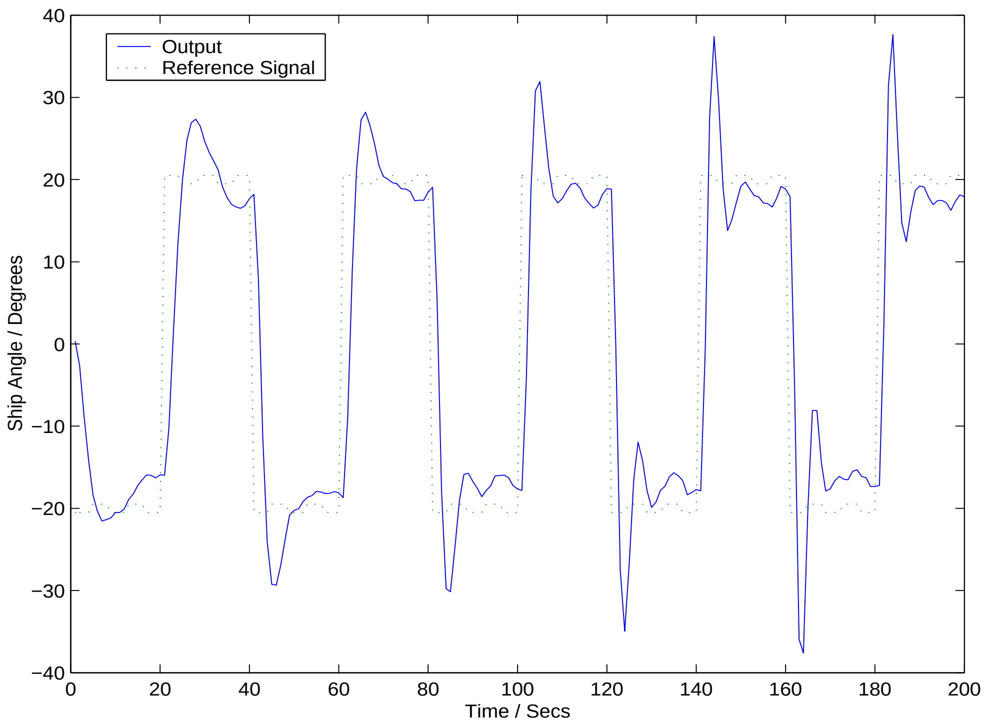

Figure 2: Reference and Ship Roll Angle

system varies every 40 seconds as would be expected from the varying natural frequency. Clearly, the ship remains stable with overshoot of no more than 40%. Figure 3 shows the 6 identified parameters, two from the plant and disturbance model denominator, a_1 and a_2, two from the plant numerator, b_1 and b_2, and two from the disturbance numerator, c_1 and c_2. It is clear that the a_1 and a_2 values are decreasing in magnitude over time and the b_1 and b_2 parameters are increasing, due to the increase in natural frequency. The outcome of these parameter variations is given in Figure 4, which shows that both proportional and derivative gains are decreasing over time. Evidently, as the natural frequency and therefore the bandwidth of the plant rises, it is necessary to decrease controller gain to avoid instability.

The system parameter estimates are held constant at guessed values for the first 6 seconds of the simulation until the recursive least squares data vector is full. The adaptive predictive control algorithm updates the PD gains every 4 seconds, as it is unnecessary to update more often for a slowly varying plant. For these reasons, control gains are held constant for the first 8 seconds, at which time the algorithm uses the latest identified system parameters in the optimisation. Clearly, the identified system parameters do not approach the correct values until after around 25 seconds. This is an indication that the algorithm is more robust than a standard self-tuning algorithm that depends upon identified parameters only. The weight of the 4 fixed models in the adaptive predictive optimisation keeps the control gains at sensible values, although there is a marked fall immediately after the algorithm 'turns on'.

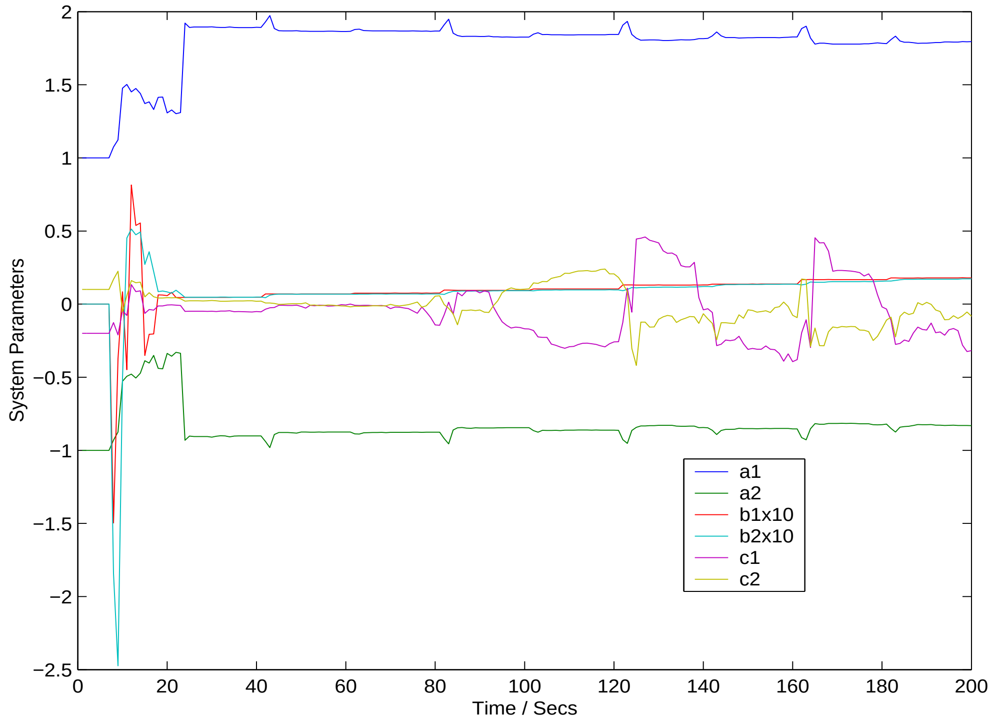

Figure 3: System Parameters

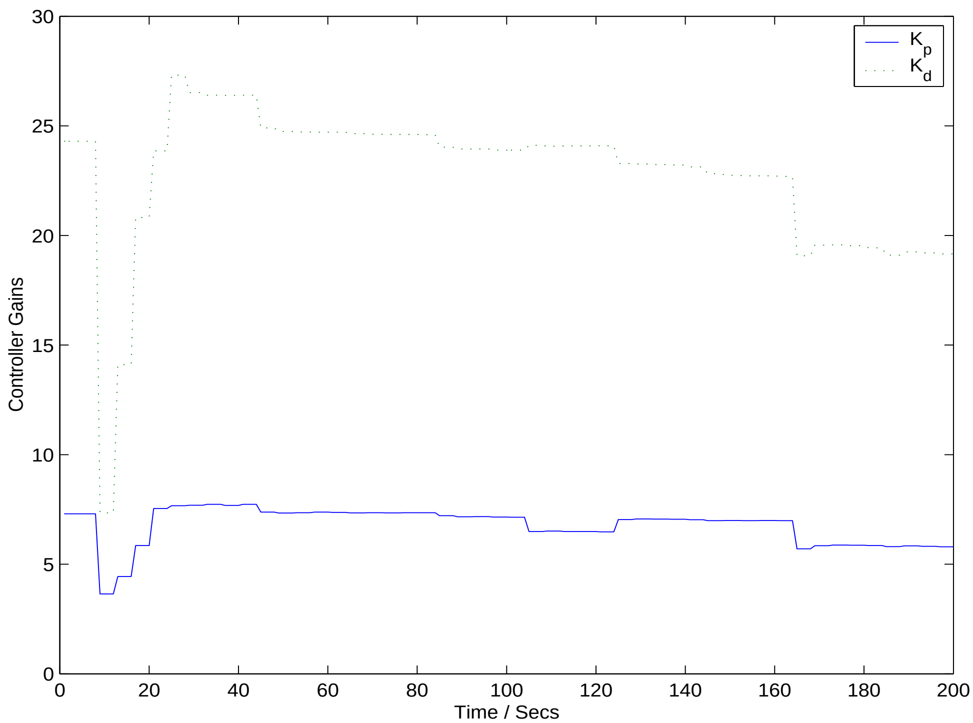

Figure 4: Controller Gains

6 Conclusions

In this paper, a novel predictive adaptive control technique has been presented. The advantage of this method is the combination of the benefits of self-tuning and multiple-model restricted-structure optimal controller designs into one scheme, as well as the incorporation of future set-point knowledge and a multi-step cost index. A self-tuner is able to adapt to changing system parameters at the expense of possible instability. A multiple-model optimal controller gives greater assurance of stability over a wide range of operating points with the expense of conservative performance. A multiple-model adaptive controller is intermediate to these two schemes. It provides a certain amount of confidence in stability, due to the weighted effect of fixed known models in the optimisation, plus a performance enhancement due to the incorporation of system identification knowledge from one sample point to the next. The restricted structure of the control law provides simplicity of implementation, and transparency of the solution to those acquainted with much-used classical control laws. The predictive aspect of the controller improves setpoint tracking ability and can produce more efficient use of actuators. To further extend and bring rigour to this work, an investigation of the convergence of the restricted structure algorithm would be desirable. Also, a criterion for the robustness of a given multiple-model problem would be beneficial.

Acknowledgement: The authors are grateful for the support of the Engineering and Physical Sciences Research Council (EPSRC) of the UK for their general support via the Platform Grant, Number GR/R04683/01.

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A Appendix

A.1 Four fixed ship roll models

$$G_{ship}(s) = \frac{\theta(s)}{\delta(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + w_n^2} \tag{A.1}$$

$\theta(s)$ - Ship Roll Angle , $\delta(s)$ - Fin Angle

$$\omega_{n1} = 0.1, \omega_{n2} = 0.125, \omega_{n3} = 0.15, \omega_{n4} = 0.175$$

$$\zeta_1 = 0.1, \zeta_2 = 0.1, \zeta_3 = 0.1, \zeta_4 = 0.1$$