Synergetic Synthesis of Nonlinear Kinematics Regulators for Mobile Robots

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Abstract

We suggest new control strategy for mobile robots. This strategy based on forming some desired behavior structure. Desired structure is introducing to control laws by using methods of synergetic theory of control.

1 Introduction

The problem of stand-alone robot systems design that are operating in extreme condition or in human life unsafely condition has a great number of subtasks. Some of these subtasks are: embodiment; sensory and navigation data processing from board sensors and devices; decision-making for further behavior under unpredictable environment changing; forming control actions for robot execution units; and so on. Important stage of mobile robot (MR) design is control system synthesis that provides stable motion along a set trajectory with desired velocity. In case of MR (in contrast to ordinal manipulation robot) we have transport module as nonholonomic system, i.e. in mathematics description there are rigid connections between generalized coordinates and external coordinates [1, 2].

2 MR Description

Three-wheeled constructions provoke a great interest because of their maneuverability and ease of assembly. The considered MR configuration is a platform with three wheels. Driving and steering is performed by the front wheel (Fig. 1) [3]. The schema shows the robot's geometric dimensions: b and R are base and wheel radius respectively. The mobile coordinate origin and rotation center is located at the point O_1.

At present control laws for MR is formed on basis of so-called kinematic schemes. If we denote the driving wheel rotation speed as ω, and α as its angle, the kinematic relations of the MR can be written as follows:

$$\begin{cases} \dot{x}(t) = R\omega\cos(\alpha)\cos(\theta), \\ \dot{y}(t) = R\omega\cos(\alpha)\sin(\theta), \\ \dot{\theta}(t) = \dfrac{R\omega\sin(\alpha)}{b}. \end{cases} \tag{2.1}$$

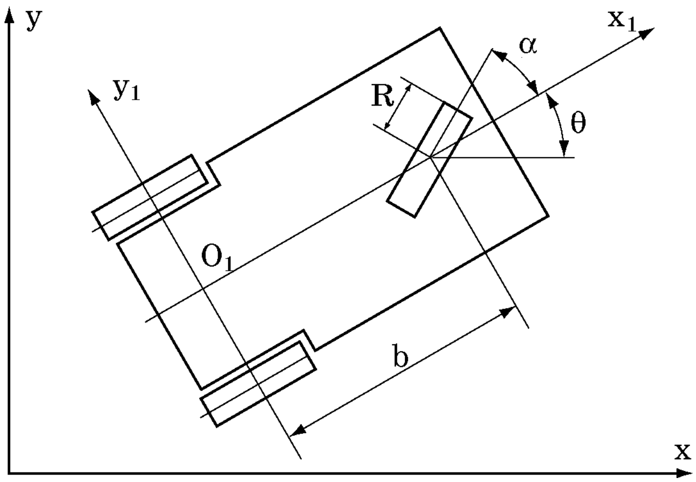

Figure 1: Kinematic schema of the mobile robot

where θ – is course angle, x, y – are Cartesian (or external) coordinates. The path speed is determined by the following relation: $R\omega\cos(\alpha)$. The system (2.1) is a kinematic description of the MR's behavior, i.e. it determines connection between angle of the steering wheel and the changes of the external coordinates and course angle.

3 Control laws synthesis

The control laws synthesis task is to ensure MR's motion along a set trajectory with a set path speed. The considered kinematic model consists of two control channels: angle $\alpha(x, y, \theta)$ and path speed $\omega(x, y, \theta)$ of the steering wheel. Let's synthesize a controller for the system (2.1) that will ensure a set type of MR coordinates changing. Let's set the desired behavior of the system in a form of system of differential equation:

$$\begin{cases} \dot{x}_1(t) = Z_1(x_1, x_2), \\ \dot{x}_2(t) = Z_2(x_1, x_2). \end{cases} \tag{3.1}$$

where $x_1(t), x_2(t)$ – are variables corresponding to the MR Cartesian coordinates.

Let's introduce the manifold for the system (2.1) [4]:

$$\psi_1 = \theta - \phi(x, y), \tag{3.2}$$

and write the following functional equation for this manifold:

$$T_1\dot{\psi}_1(t) + \psi_1 = 0, \tag{3.3}$$

where $T_1 > 0$.

If ψ_1 and derivative of ψ_1 to substitute to (3.3) we get:

$$T_1\left(\frac{R\omega\sin(\alpha)}{b} - \dot{\phi}_t(x,y)\right) + \theta - \phi(x,y) = 0. \tag{3.4}$$

Let's find $\sin(\alpha)$ from the equation (3.4):

$$\sin(\alpha) = \frac{b}{T_1 R\omega}(\phi(x,y) - \theta) + \frac{\dot{\phi}_t(x,y)b}{R\omega}, \tag{3.5}$$

i.e. $\sin(\alpha) = f(\phi(x,y), \dot{\phi}_t(x,y))$.

After the system gets to the manifold ψ_1, the expression for ω takes the following form:

$$\sin(\alpha_{\psi_1}) = \frac{\dot{\phi}_t(x,y)b}{R\omega},$$

According to principle of phase flow contraction [4] system (2.1) is decomposed on the manifold ψ_1. Accounting that $\cos(\alpha_{\psi_1}) = \sqrt{1-\sin^2(\alpha_{\psi_1})} = \sqrt{1-\left(\frac{\dot{\phi}_t(x,y)b}{R\omega}\right)^2}$, we get:

$$\begin{cases} \dot{x}_{\psi_1}(t) = R\omega\sqrt{1-\left(\dfrac{\dot{\phi}_t(x,y)b}{R\omega}\right)^2}\cos(\phi(x,y)), \\ \dot{y}_{\psi_1}(t) = R\omega\sqrt{1-\left(\dfrac{\dot{\phi}_t(x,y)b}{R\omega}\right)^2}\sin(\phi(x,y)). \end{cases}$$

Equaling the obtained system to (3.1) yields:

$$\begin{cases} R\omega\sqrt{1-\left(\dfrac{\dot{\phi}_t(x,y)b}{R\omega}\right)^2}\cos(\phi(x,y)) = Z_1(x_1,x_2), \\ R\omega\sqrt{1-\left(\dfrac{\dot{\phi}_t(x,y)b}{R\omega}\right)^2}\sin(\phi(x,y)) = Z_2(x_1,x_2). \end{cases}$$

Solving this system we get:

$$\phi(x_1,x_2) = \arctan\left(\frac{Z_2(x_1,x_2)}{Z_1(x_1,x_2)}\right), \omega = \pm\sqrt{\frac{Z_2^2(x_1,x_2)}{\sin^2(\phi(x_1,x_2))R^2} + \frac{\dot{\phi}_t^2(x_1,x_2)b^2}{R^2}}. \tag{3.6}$$

The time derivative of the function $\phi(x_1,x_2)$ is given by the following expression:

$$\dot{\phi}_t(x_1,x_2) = \frac{\partial \phi(x_1,x_2)}{\partial x_1} Z_1(x_1,x_2) + \frac{\partial \phi(x_1,x_2)}{\partial x_2} Z_2(x_1,x_2) =$$

$$= \frac{\frac{\partial Z_2(x_1,x_2)}{\partial x_1} Z_1^2(x_1,x_2) - Z_1(x_1,x_2) Z_2(x_1,x_2) \left(\frac{\partial Z_2(x_1,x_2)}{\partial x_2} - \frac{\partial Z_1(x_1,x_2)}{\partial x_1} \right)}{Z_1^2(x_1,x_2) + Z_2^2(x_1,x_2)} -$$

$$- \frac{\frac{\partial Z_1(x_1,x_2)}{\partial x_2} Z_2^2(x_1,x_2)}{Z_1^2(x_1,x_2) + Z_2^2(x_1,x_2)}.$$

So the equation for the front wheel takes the following form (basing on the physical meaning we take the positive value of speed):

$$\omega = \sqrt{\frac{Z_2^2(x_1,x_2)}{\sin^2(\phi(x_1,x_2))R^2} + \frac{\dot{\phi}_t^2(x_1,x_2)b^2}{R^2}}, \tag{3.7}$$

The equation for the front wheel angle we express from (6):

$$\alpha = \arcsin\left(\frac{b}{T_1 R \omega} (\phi(x_1,x_2) - \theta) + \frac{\dot{\phi}_t(x_1,x_2) b}{R \omega} \right). \tag{3.8}$$

The set MR's behavior (3.1) we present in the form of functions that separate behavior MR on two regimes.

1) Motion out of certain trajectory $f(x_1,x_2)$:

$$\begin{cases} Z_1^{(1)}(x_1,x_2) = -f(x_1,x_2) \arctan\left(\frac{\partial f(x_1,x_2)}{\partial x_1} \right), \\ Z_2^{(1)}(x_1,x_2) = -f(x_1,x_2) \arctan\left(\frac{\partial f(x_1,x_2)}{\partial x_2} \right). \end{cases} \tag{3.9}$$

i.e. while MR moves far from trajectory $f(x_1,x_2)$ it runs towards desired one with arbitrary contour speed (we use a particular case of the gradient search method).

2) Motion along a set trajectory $f(x_1,x_2)$:

$$\begin{cases} Z_1^{(2)}(x_1,x_2) = V_k \sin(\xi), \\ Z_2^{(2)}(x_1,x_2) = V_k \cos(\xi), \end{cases} \tag{3.10}$$

i.e. while MR moves along trajectory $f(x_1,x_2)$ it contour speed remains constant. Value of the speed determines from:

$$\sqrt{\dot{x}_1^2(t) + \dot{x}_2^2(t)} = \sqrt{V_k^2 \sin^2(\xi) + V_k^2 \cos^2(\xi)} = V_k.$$

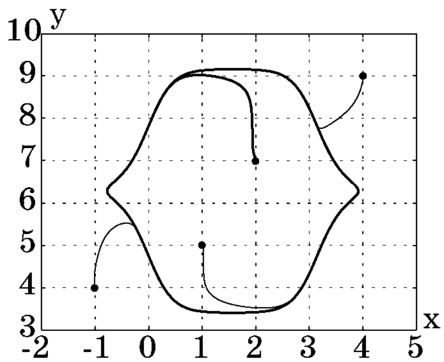

Figure 2: MR's Trajectory

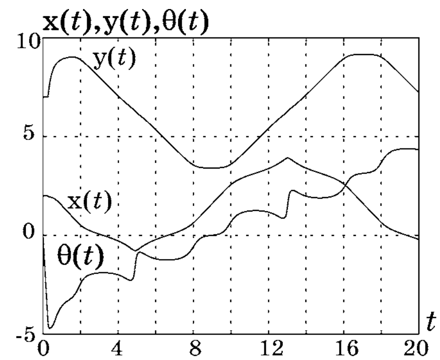

Figure 3: MR's external coordinates

The parameter ξ – in the system (3.10) set the angle of the trajectories tangent line in each point and is given by the following expression:

$$\xi = \arctan\left(\frac{\frac{\partial f(x_1,x_2)}{\partial x_2}}{\frac{\partial f(x_1,x_2)}{\partial x_1}}\right).$$

Let's introduce function $\Phi(\delta) = \left(1 - \frac{4}{\pi^2}\arctan^2(k\delta)\right)$ that will separate regimes of MR's motion. Therefore, we obtain desired behavior of MR:

$$\begin{cases} Z_1(x_1,x_2) = Z_1^{(2)}(x_1,x_2)\Phi(f(x_1,x_2)) - Z_1^{(1)}(x_1,x_2), \\ Z_2(x_1,x_2) = Z_2^{(2)}(x_1,x_2)\Phi(f(x_1,x_2)) - Z_2^{(1)}(x_1,x_2), \end{cases} \tag{3.11}$$

After mathematical conversion system (3.11) takes the following form:

$$\begin{aligned} \dot{x}_1(t) &= \frac{Dfx_2V_k}{s}\left(1 - \frac{4}{\pi^2}\arctan^2(kf(x_1,x_2)) - f(x_1,x_2)\arctan(Dfx_1)\right), \\ \dot{x}_2(t) &= \frac{-Dfx_1V_k}{s}\left(1 - \frac{4}{\pi^2}\arctan^2(kf(x_1,x_2)) - f(x_1,x_2)\arctan(Dfx_2)\right). \end{aligned} \tag{3.12}$$

where $Dfx_1 = \frac{\partial f(x_1,x_2)}{\partial x_1}$, $Dfx_2 = \frac{\partial f(x_1,x_2)}{\partial x_2}$, $s = \sqrt{(Dfx_1)^2 + (Dfx_2)^2}$.

Let's perform simulations for closed-loop system with the following parameters: $f(x_1,x_2) = \tanh(4\sin(x_1)) + \cos(x_2) - 0.03$, $T_1 = 0.04$, $V_k = 1$, $k = 10$, $b = 1$, $R = 2$. The simulations results with the following initial conditions $\{x(0) = -1, y(0) = 4, \theta(0) = 0; x(0) = 1, y(0) = 5, \theta(0) = 0; x(0) = 2, y(0) = 7, \theta(0) = 0; x(0) = 4, y(0) = 9, \theta(0) = 0\}$ are presented in fig. 2–5.

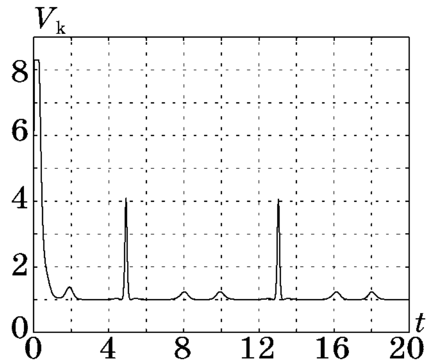

Figure 4: Path Speed

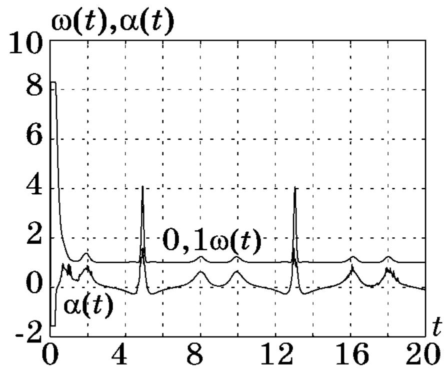

Figure 5: Control actions

4 Conclusion

The mentioned synergetic synthesis method based on building some functional dependence for control actions. These dependences are building by using desired form of motion trajectories. Obtained control lows guarantee maintenance of constant trajectory speed only when MR moves along set trajectory. Such approach improves maneuverability of MR when it moves out of trajectory.

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