Performance Enhancement of Controlled Diffusion Processes by Moving Actuators

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Abstract

The present research work deals with the systematic development and implementation of a practical algorithm for an actuator activation and control policy through a scheme of moving actuators for systems governed by parabolic partial differential equations (PDEs). Systems of parabolic PDEs typically describe diffusion and other transport processes often encountered in a multitude of industrial applications. Under the proposed algorithmic scheme, one way to view the system under consideration is to assume that it has multiple actuators and it is desired to activate only one such actuator during a given time interval while the remaining actuators remain dormant. The same algorithm can also be applied to a system with a single actuator capable of moving at a priori selected positions within the spatial domain. Standard state feedback controller synthesis methods based on linear matrix inequality-techniques (LMIs) are employed for a finite-dimensional Galerkin approximation of the original distributed parameter system, and the value of an appropriately selected objective function (performance index/functional) is explicitly calculated by solving a location-parameterized Lyapunov matrix equation. On the basis of the aforementioned explicit characterization of the objective function values, a systematic optimization algorithm can be developed that offers a transparent guidance policy and optimal switching rules between the various actuator positions for performance enhancement purposes. An illustrative example with simulation results of an 1-D diffusion process is included to support the paper's theoretical findings and evaluate the performance-enhancing capabilities of the proposed scheme in a typical industrial process such as the one considered in the present study.

1 Introduction

The role of actuator selection in the overall system performance has been recognized as an important design component in many control systems, see for example the survey paper by van de Wal and de Jager [25]. The number, type and placement of both actuator and sensors received considerable attention primarily by researchers working on the control of flexible space structures, acoustic cavities and transport-reaction processes [1, 2, 12, 13, 14, 17, 21, 22, 23, 24]. Using performance or controllability/observability measures, the actuating and sensing devices were permanently mounted on the host structure, cavity or chemical reactor according to the optimization measure used. While optimal versus non-optimal actuator and/or sensor location yielded improved and noticeable performance, it nonetheless ignored the effects of the spatiotemporal variability of the exogenous inputs in all the above approaches. For example, in the control of flexible structures or chemical reactors, time-varying disturbances might enter at different sections of the spatial domain at different time intervals, and hence an actuator closer to the disturbance would certainly have more control authority than an actuator far away from the "local" disturbance. Using this idea of utilizing "local" actuators that can have increased authority at different segments of the spatial domain over different time intervals, we take into account the spatiotemporal variability of disturbances and exogenous inputs and propose an actuator switching scheme which enhances the closed loop performance. From a technical point of view, standard Galerkin methods are employed that lead to an accurate finite-dimensional approximation of the original distributed parameter system, on the basis of which, local state feedback control laws are derived for a prespecified set of actuator positions using well-known LMI techniques [3, 20]. A physically meaningful quadratic performance functional/index or objective/cost function is considered, whose value is explicitly calculated and optimized over a short time-horizon via the solution of an appropriately parameterized Lyapunov matrix equation and with respect to the set of actuator positions considered above. As time progresses, the above computational steps are repeated over a sequence of "time-windows", giving rise to a transparent algorithmic procedure that results in a practical guidance policy and "optimal" switching rules for the scheme of moving actuators considered. Finally, the proposed algorithm is evaluated in an illustrative case study, where its performance-enhancing characteristics are clearly demonstrated through simulation studies.

2 Modeling Equations

We consider the 1-D controlled diffusion equation

$$\begin{aligned} \frac{\partial x}{\partial t} &= \frac{\partial}{\partial \xi}\left(\kappa(\xi)\frac{\partial x}{\partial \xi}\right) + b(\xi)u(t) + d(\xi)w(t), \\ x(t,0) &= 0 = x(t,\ell), \\ x(0,\xi) &\in L^2(0,\ell), \end{aligned} \tag{2.1}$$

where $x(\xi,t)$ denotes the state, $\xi \in \Omega = [0,\ell] \subset \mathbb{R}$ is the spatial coordinate, $t \in [t_0,\infty)$ is the time variable, $u(t)$ denotes the control signal, $b(\xi)$ denotes the spatial distribution of the actuating device, $w(t)$ the unknown exogenous input signal and $d(\xi)$ the spatial distribution of the disturbance. The state space in this case is $\mathcal{X} = L^2(0,\ell)$ with the standard L^2 inner product and norm denoted by $\langle \cdot, \cdot \rangle_{\mathcal{X}}$ and $\| \cdot \|_{\mathcal{X}}$ respectively. The control objective is to choose the signal $u \in L^2([t_0, \infty); \mathbb{R})$ so that regulation of the state $x(t, \xi)$ to zero is achieved while a certain cost functional which penalizes the total energy is minimized. It is assumed that the spatial distribution of the actuating device is spanned (locally) over a portion of the spatial domain and is given by

$$b(\xi_0) = \begin{cases} \dfrac{1}{2\epsilon} & \text{if} \qquad \xi_0 - \epsilon \leq \xi \leq \xi_0 + \epsilon \\ 0 & \text{otherwise.} \end{cases}$$

Notice, that the above approximation for $b(\xi)$ avoids any regularity problems due to the unbounded nature of a pointwise (in space) actuator distribution (i.e. a spatial delta function). Furthermore, one must ensure that the location ξ_0 of the actuator is such that *approximate controllability* of (2.1) is ensured [4].

The above system can be placed in an abstract setting written as an evolution system [5, 15, 16]

$$\dot{x}(t) = \mathcal{A}x(t) + \mathcal{B}u(t) \tag{2.2}$$

in the state space $\mathcal{X}$. Indeed, under the above representation the system operator attains the following form

$$\mathcal{A}\phi = \frac{d}{d\xi}\left(\kappa(\xi)\frac{d}{d\xi}\right)\phi,$$

with domain

$$D\text{om}(\mathcal{A}) = \left\{\psi \in L^2(0,\ell) \,\Big|\, \psi, \frac{d\psi}{d\xi} \text{ are abs. continuous}, \frac{d^2\psi}{d\xi^2} \in L^2(0,\ell) \text{ and } \psi(0) = 0 = \psi(\ell)\right\}$$

and the input operator by

$$\mathcal{B}u(t) = b(\xi)u(t), \quad \mathcal{B} \in \mathcal{L}(\mathbb{R}, \mathcal{X}).$$

Drawing from already established results in the pertinent systems literature [4, 15, 18], a state feedback controller that would minimize an associated LQR functional of the form

$$J(x_0; u) = \int_{t_0}^{\infty} \left[\langle x, Qx \rangle_{\mathcal{X}} + ru^2\right] \, dt, \tag{2.3}$$

can be synthesized, where $r > 0$ is a suitably chosen "weight-factor" and Q is a coercive operator. The cost functional (2.3) is finite for a square integrable control input since the diffusion system in question is optimizable as a consequence of its exponential stabilizability [4]. In this case one solves the Operator Algebraic Riccati Equation (OARE)

$$\langle \mathcal{A}\phi, \mathcal{P}\psi \rangle_{\mathcal{X}} + \langle \phi, \mathcal{P}\mathcal{A}\psi \rangle_{\mathcal{X}} + \langle Q\phi, \psi \rangle_{\mathcal{X}} + \langle \mathcal{P}\mathcal{B}r^{-1}\mathcal{B}^*\mathcal{P}\phi, \psi \rangle_{\mathcal{X}} = 0 \tag{2.4}$$

for $\phi, \psi \in D\text{om}(\mathcal{A})$. The optimal control signal can be proven to be [4]

$$u(t) = -r^{-1}\mathcal{B}^*\mathcal{P}x(t). \tag{2.5}$$

and the resulting optimal value of the cost functional is given by

$$J^*(x_0; u) = \langle x(t_0), \mathcal{P}x(t_0)\rangle_{\mathcal{X}}. \tag{2.6}$$

For a given operator Q and a fixed value of r, one may further enhance the closed loop performance by finding an optimal location ξ_a of the actuating device, in the sense of minimizing $J^*(x_0; u)$. Usually, there is a finite set of candidate actuator positions and hence one may optimize the cost value $J^*(x_0, u)$ over this set of actuator locations [18]. Going even further, one may utilize a finite set of m actuating devices placed (or mounted) in "optimal" locations in the spatial domain, and activate one such a device over a time window while the remaining $(m-1)$ devices are kept dormant (or inactive). This procedure may be repeated over different time intervals. Hence, one arrives at a *switched* system, where, in addition to the control signal, the location of the actuating device also changes over a given time interval. A measure for choosing which actuator is to be activated over a certain time interval and what the control signal should be for this interval, was initially proposed in [11, 26] for thermal processes and was later verified experimentally in [19]. The same scheme was applied to the control of flexible structures in [6, 7, 8, 10]. This scheme is essentially based on LQR measures and at each time interval, the *LQR cost-to-go* was re-evaluated for each of the finite locations of the actuators. The location that yielded the smallest value of the optimal LQR cost was the one used by the moving actuator. In this scheme, both the actuator position and feedback gain were changing at each time interval thus imposing a heavy computational load. It should be noted that while the aforementioned research efforts dealt with a single actuating device capable of moving at m pre-selected positions in the spatial domain, they are nonetheless applicable to the current case of having m available actuators in which only one is active while the remaining $(m-1)$ remain dormant over a certain time interval. Variants of this approach were presented in [9] in which LMI's were utilized to calculate a *common* feedback gain that was used in each of the active actuators.

3 Actuator Activation Policy - Switching Rules

In the present section, a new systematic algorithm will be presented through which a practical actuator guidance policy is realized that activates a single actuator out of m available ones. In particular, the temporal scheduling pattern and the switching rules between the occasionally activated actuator and the $(m-1)$ dormant ones along the system's spatial domain are based on transparent optimality criteria that reflect key closed-loop performance characteristics. It should be emphasized that unlike previous research efforts, the current algorithm only switches (changes) the actuators while keeping the *same* feedback gain as it is computed through standard LMI-based controller synthesis methods [3]. Notice, that the fundamental difference between the proposed integrated *fixed-structure/fixed-gain* LMI-based controller synthesis and actuator switching scheme, and previous approaches [8, 7, 26, 6], lies in the solution of the infinite-dimensional optimal control problem that inevitably resulted in spatiotemporal variations of the feedback gains computed. Thus, irrespective of the actuator used by the system, the control signal remains the *same* in the proposed approach, significantly reducing the computational load associated with a direct computer-aided implementation of the control policy.

Let us now rewrite system (2.1) in its standard state-space dynamic evolution representation form

$$\dot{x}(t) = \mathcal{A}x(t) + \mathcal{B}(\xi)u(t), \tag{3.1}$$

where now the input operator is explicitly parameterized in terms of its location. In the present study, a finite set of m actuator locations

$$\Theta = \{\xi_1, \xi_2, \ldots, \xi_m\}, \tag{3.2}$$

is considered. Each of the candidate locations $\xi_i \in \Theta$ is such that approximate controllability is guaranteed, i.e. the pair $(\mathcal{A}, \mathcal{B}(\xi_i))$ is approximately controllable for all $\xi_i \in \Theta$. Following [4], for the case of spatially invariant diffusion processes, one may choose the locations $\xi_i \in \Theta$ such $\sin(n\pi\xi_i/\ell) \neq 0$ for $n \geq 1$. For each element in Θ, one can find a feedback gain that is *common* to all these elements. Specifically, one can solve the following *Linear Operator Inequality*

$$\langle(\mathcal{A} + \mathcal{B}(\xi_i)\mathcal{K})\phi, \mathcal{S}\psi\rangle_{\mathcal{X}} + \langle\mathcal{S}\phi, (\mathcal{A} + \mathcal{B}(\xi_i)\mathcal{K})\psi\rangle_{\mathcal{X}} < 0, \tag{3.3}$$

for $\phi, \psi \in D\mathrm{om}(\mathcal{A})$ and all $\xi_i \in \Theta$, where $\mathcal{K}$ is the unknown *common* feedback gain. In this case, one may easily conclude that the feedback control law $u(t) = \mathcal{K}x(t)$ is a stabilizing pole-placing one irrespective of the actuator used. Equivalently, the closed-loop operator $\mathcal{A}_i \triangleq \mathcal{A} + \mathcal{B}(\xi_i)\mathcal{K}$ has the desirable eigenspectrum and generates an exponentially stable C_0 semigroup for all ξ_i, $i = 1, 2, \ldots, m$.

The problem under consideration can now be stated as follows:

Problem Statement: *For each time interval of fixed length Δt, develop an actuator guidance policy and algorithm that uniquely determines which actuator out of the m available ones will stay active and which ones are to stay dormant.* In this case, the control signal will always be the same and given by $u = \mathcal{K}x$, where $\mathcal{K}$ satisfies (3.3), *but* the actuating device will be changing at the beginning of each time interval.

In order to address the problem stated above, the following switching policy for performance enhancement is proposed:

Algorithm: Guidance policy

(1) Solve the *location-parameterized Operator Lyapunov Equation*

$$\boxed{\langle \mathcal{A}_i\phi, \mathcal{P}(\xi_i)\psi\rangle_{\mathcal{X}} + \langle \mathcal{P}(\xi_i)\phi, \mathcal{A}_i\psi\rangle_{\mathcal{X}} = -\langle \phi, \mathcal{Q}\psi\rangle_{\mathcal{X}}, \qquad \phi, \psi \in \mathrm{Dom}(\mathcal{A})} \tag{3.4}$$

to obtain the m Lyapunov solutions $\mathcal{P}(\xi_i)$ that correspond to each of the actuator locations ξ_i.

(2) For each time interval $I_k = [t_k, t_k + \Delta t)$ where k is viewed as the discrete-time index, identify the actuator that minimizes the *cost-to-go* quadratic functional reflecting standard closed-loop performance criteria

$$\boxed{J(\xi_j; t_k) = \int_{t_k}^{\infty} \langle x, \mathcal{Q}x\rangle_{\mathcal{X}}\, dt.} \tag{3.5}$$

Notice, that the value of the above performance index can be explicitly calculated using the following formula:

$$J(\xi_j; t_k) = \langle x(t_k), \mathcal{P}(\xi_j)x(t_k)\rangle_{\mathcal{X}} \tag{3.6}$$

where $\mathcal{P}(\xi_j)$ is the location-patameterized solution of (3.4) and $x(t_k)$ denotes the initial condition in the interval I_k. Therefore, for the time-interval I_k, the actuator position ξ^* that minimizes the *cost-to-go* functional is given by

$$\boxed{\begin{aligned} \xi^* &= \arg\min_{\xi_j \in \Theta} J(\xi_j; t_k) \\ &= \arg\min_{\xi_j \in \Theta} \langle x(t_k), \mathcal{P}(\xi_j)x(t_k)\rangle_{\mathcal{X}} \end{aligned}} \tag{3.7}$$

and the corresponding actuator that stays active over I_k is uniquely determined.

(3) Steps 1 and 2 are repeated for the next time-interval $I_{k+1} = [t_{k+1}, t_{k+1} + \Delta t)$.

Remark 3.1. *In the above actuator guidance policy, the durations of each of the time windows I_k, $k = 1, 2, \ldots,$ were assumed identical. The value of Δt was not optimized, but instead was assumed of constant value that was dictated by hardware considerations and time constants of the individual systems.*

Remark 3.2. *The existence of the self-adjoint solution to the m operator algebraic Lyapunov equations* (3.4) *is ensured by the exponential stability of the infinitesimal generators $\mathcal{A}_i$, $i = 1, 2, \ldots, m$ and the approximate observability of the pairs $(\mathcal{A}_i, \mathcal{Q}^{\frac{1}{2}})$ with $\mathcal{Q}^{\frac{1}{2}}$ being the square root of the coercive operator $\mathcal{Q}$ [4].*

4 Implementation Aspects of the Proposed Algorithm

The proposed actuator guidance policy, as well as the corresponding switching rules outlined previously, reflect the flow of key ideas that enabled the formulation and algorithmic treatment of the problem of interest to be realized through the use of an abstract operator-based language of analysis that is deemed appropriate for distributed parameter systems. From a practical point of view however, the proposed algorithmic framework needs to be implemented in a more comprehensive manner and with the aid of a computer. Indeed, since the proposed algorithm requires the solution to an operator Lyapunov equation (3.4) along with the operator inequalities (3.3), one first approximates the infinite dimensional system (3.1) using an exponential stabilizability-preserving approximation scheme in order to arrive at a matrix system

$$\dot{x}^n(t) = Ax^n(t) + B(\xi)u^n(t).$$

The set of linear matrix inequalities (LMIs) corresponding to (3.3)

$$(A + B(\xi_i)K)^T \Sigma + \Sigma (A + B(\xi_i)K) < 0$$

are then solved in order to find the finite dimensional aproximation K of the common feedback gain $\mathcal{K}$. Lastly, the location-parameterized Lyapunov equation

$$(A + B(\xi_i)K)^T P(\xi_i) + P(\xi_i) (A + B(\xi_i)K) = -Q$$

is solved for each location $\xi_i \in \Theta$ using the finite dimensional representation of (3.7). The criterion (3.6) for the actuator policy is then replaced by its finite dimensional approximation

$$J^n(\xi_j; t_k) = (x^n(t_k))^T P(\xi_j)x^n(t_k),$$

which along with the finite dimensional control signal $u^n(t) = Kx^n(t)$ comprise the actuator quidance and control policies implemented in the system.

The above algorithmic scheme provides a practical answer to the actuator guidance policy problem stated before, leading to an easily implementable set of switching rules between various actuator locations on the basis of closed-loop performance-related optimality measures. The performance-enhancing characteristics of the proposed algorithm are illustrated in the next section's case study through numerical simulations.

5 Numerical Results

For our numerical investigation, the following PDE is considered

$$\frac{\partial x}{\partial t} = \kappa \frac{\partial^2 x}{\partial \xi^2} + b(\xi)u(t) + d_1(\xi)w(t) + d_2(\xi)w(t - \frac{5}{2}) + d_3(\xi)w(t + \frac{5}{2}).$$

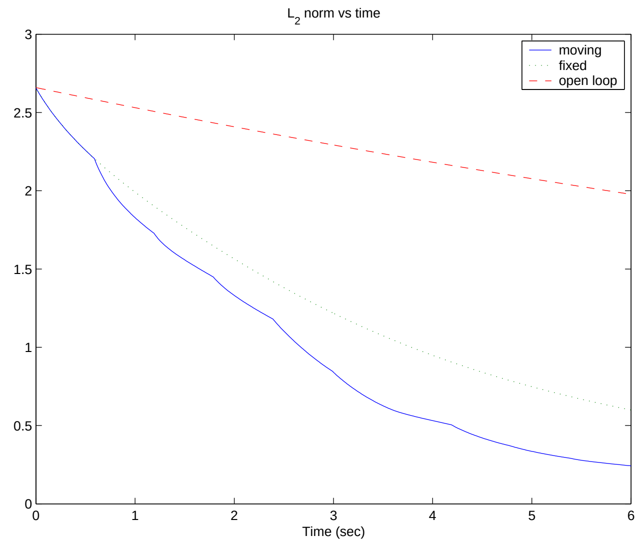

Figure 1: Evolution of L^2 norm; open loop (dashed), fixed actuator (dotted) and moving actuator (solid).

As an initial condition, we considered $x(0,\xi) = 10\sin(\pi\xi)$ and a spatially invariant thermal diffusivity $\kappa = 0.01$. Six possible actuator locations were considered at the locations $\xi_i = (i - 0.345) * \ell/6$, $i = 1, 2, \ldots, 6$. The disturbance was taken as $w(t) = 0.005\sin(\frac{\pi t}{5})$, and the spatial distributions of the disturbances were given by $d_1(\xi) = \chi_{[0.2,0.3]}(\xi)$, $d_2(\xi) = \chi_{[0.4,0.6]}(\xi)$ and $d_3(\xi) = \chi_{[0.8,0.9]}(\xi)$, where $\chi_{[\alpha,\beta]}(\xi)$ denotes the characteristic function in the interval $[\alpha, \beta]$, ie $\chi_{[\alpha,\beta]}(\xi) = 1$ if $\alpha \leq \xi \leq \beta$ and $\chi_{[\alpha,\beta]}(\xi) = 0$ if $\xi \in [0,\ell] \setminus [\alpha,\beta]$. The system was simulated for $t_f = 6$ seconds and actuator switching was implemented at every $t_f/10 = 0.6$ seconds. Figure 1 depicts the $L^2(0,\ell)$ state norms for the open loop case (dashed), the case of a fixed actuator (dotted) and the case of a switching actuator (solid). It is observed that when a switching actuator is utilized, the closed loop performance is enhanced, as this is revealed by the faster convergence of the state norm to zero. The temperature distribution is depicted in Figure 2 for two different time instances. One may observe that pointwise-in-space convergence of the state is improved when a switching actuator is utilized. The activation sequence is depicted in Figure 3.

6 Concluding Remarks

A new systematic and practical algorithm was developed that results in a transparent moving actuator guidance policy and a concrete set of switching rules for performance enhancement of controlled diffusion processes. The proposed approach introduced a comprehensive *fixed-structure/fixed-gain* LMI-based controller synthesis method where the control signal remains the *same* for all actuator locations, thus significantly reducing the computational load associ-

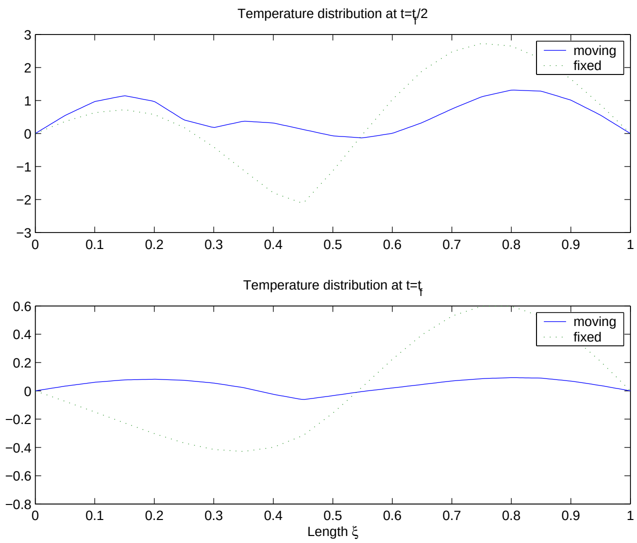

Figure 2: Spatial distribution (a) at $t = t_f/2$ and (b) at $t = t_f$m; fixed actuator (dotted) and moving actuator (solid).

ated with other approaches in the literature.The temporal scheduling pattern and switching rules between the intermittently activated actuator and the remaining dormant ones was based on straightforward optimality criteria that reflect key closed-loop performance characteristics. In particular, the value of an appropriately selected performance functional was explicitly calculated via a location-parameterized Lyapunov matrix equation, and subsequently optimized with respect to the set of actuator locations. Finally, the performance-enhancing attributes of the proposed algorithmic framework were illustrated in a representative case study by conducting numerical simulations.

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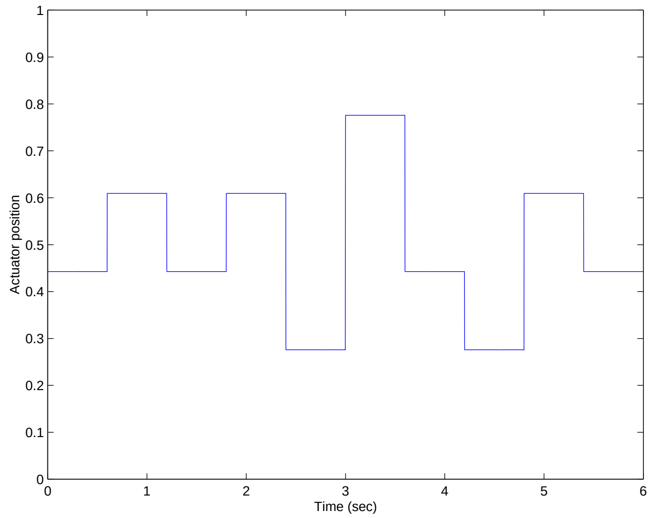

Figure 3: Actuator activation sequence.

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