Reduction of the number of parameters for all stabilizing controllers

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Abstract

This paper presents a parameterization method of stabilizing controllers that needs smaller number of parameters than previous. The result in this paper will not assume the existence of the coprime factorizability and not employ the Youla parameterization.

1 Introduction

Once the existence of the doubly coprime factorization comes to be known, it is easy to parameterize all stabilizing controllers by Youla parameterization (also called Youla-Kučera-parameterization)[2, 10, 14, 15, 16]. On the other hand, some models of control systems do not know whether or not a stabilizable plant in the class always has its doubly coprime factorization. The multidimensional systems with structural stability is one of such models[5, 4]. Further it is known that there are models such that some stabilizable plants do not have coprime factorizations[1].

In order to parameterize the stabilizing controllers of such models, the author has recently presented a parameterization method that can be applied even to the plant that has no doubly coprime factorization[7]. However the method needs a large number of parameters.

The objective of this paper is to present an alternative parameterization method of stabilizing controllers without the coprime factorizability that needs smaller number of parameters than previous. The result obtained in this paper is a unification of Youla parameterization and the method given in [7].

The approach we use in this paper is the coordinate-free approach[11, 12, 13]. The coordinate-free approach is a factorization approach[2, 15] without the coprime factorizability.

2 Coordinate-Free Approach

First we briefly introduce the notion we use, that is, the coordinate-free approach.

Sule in [12, 13] presented a theory of the feedback stabilization of multi-input multi-output strictly causal plants over commutative rings with some restrictions. This approach to the stabilization theory is called "coordinate-free approach"[11] in the sense that the coprime factorizability of transfer matrices is not required.

Let $\mathcal{R}$ denote an unspecified commutative ring. The total ring of fractions of $\mathcal{R}$ is denoted by $\mathcal{F}(\mathcal{R})$; that is, $\mathcal{F}(\mathcal{R}) = \{n/d \,|\, n, d \in \mathcal{R},\ d$: nonzerodivisor$\}$.

We consider that *the set of stable causal transfer functions* is a commutative ring. Throughout the paper, the set of stable causal transfer functions is denoted by $\mathcal{A}$. The total ring of fractions of $\mathcal{A}$ is denoted by $\mathcal{F}(\mathcal{A})$ or simply $\mathcal{F}$; that is, $\mathcal{F}(\mathcal{A}) = \mathcal{F} = \{n/d \,|\, n, d \in \mathcal{A},\ d$: nonzerodivisor$\}$. This is considered as *the set of all possible transfer functions*. Let $\mathcal{Z}$ be a prime ideal of $\mathcal{A}$ with $\mathcal{Z} \neq \mathcal{A}$ such that $\mathcal{Z}$ includes all zerodivisors. Define the subsets $\mathcal{P}$ and $\mathcal{P}_{\mathrm{S}}$ of $\mathcal{F}(\mathcal{A})$ as follows:

$$\mathcal{P} = \{a/b \in \mathcal{F} \,|\, a \in \mathcal{A},\ b \in \mathcal{A}\backslash\mathcal{Z}\},\ \ \mathcal{P}_{\mathrm{S}} = \{a/b \in \mathcal{F} \,|\, a \in \mathcal{Z},\ b \in \mathcal{A}\backslash\mathcal{Z}\}.$$

Then every transfer function in $\mathcal{P}$ $(\mathcal{P}_{\mathrm{S}})$ is called *causal* (*strictly causal*). Analogously, if every entry of a transfer matrix is in $\mathcal{P}$ $(\mathcal{P}_{\mathrm{S}})$, the transfer matrix is called *causal* (*strictly causal*).

Matrices A and B over $\mathcal{R}$ are *right-* (*left-*)*coprime over* $\mathcal{R}$ if there exist matrices X and Y over $\mathcal{R}$ such that $XA + YB = E$ $(AX + BY = E)$ holds. Further, an ordered pair (N, D) of matrices N and D over $\mathcal{R}$ is said to be a *right-coprime factorization over* $\mathcal{R}$ of P if (i) D is nonsingular over $\mathcal{R}$, (ii) $P = ND^{-1}$ over $\mathcal{F}(\mathcal{R})$, and (iii) N and D are right-coprime over $\mathcal{R}$. As the parallel notion, the *left-coprime factorization over* $\mathcal{R}$ of P is defined analogously.

Let $M_r(X)$ denote the $\mathcal{R}$-module generated by rows of a matrix X over $\mathcal{R}$. Let $X = AB^{-1}$ be a matrix over $\mathcal{F}(\mathcal{R})$, where A, B are matrices over $\mathcal{R}$. It is known that $M_r([\,A^t \quad B^t\,]^t)$ is unique up to an isomorphism with respect to any choice of fractions AB^{-1} of X (Lemma 2.1 of [9]). Therefore, for a matrix X over $\mathcal{R}$, we denote by $\mathcal{T}_{X,\mathcal{R}}$ the module $M_r([\,A^t \quad B^t\,]^t)$.

The stabilization problem considered in this paper follows that of Sule in [12], and Mori and Abe in [9], who consider the feedback system Σ [14, Ch.5, Figure 5.1] as in Figure 1. For further details the reader is referred to [9, 12, 14]. Throughout the paper, the plant we consider has m inputs and n outputs, and its transfer matrix, which is also called a *plant* itself simply, is denoted by P and belongs to $\mathcal{F}^{n\times m}$. We can always represent P in the form of a fraction $P = ND^{-1}$ $(P = \widetilde{D}^{-1}\widetilde{N})$, where $N \in \mathcal{A}^{n\times m}$ $(\widetilde{N} \in \mathcal{A}^{n\times m})$ and $D \in \mathcal{A}^{m\times m}$ $(\widetilde{D} \in \mathcal{A}^{n\times n})$ with nonsingular D

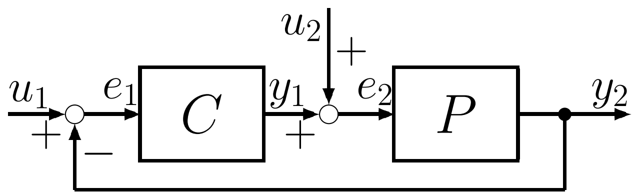


Figure 1: Feedback system Σ.

$(\widetilde{D})$.

For $P \in \mathcal{F}^{n\times m}$ and $C \in \mathcal{F}^{m\times n}$, a matrix $H(P,C) \in \mathcal{F}^{(m+n)\times(m+n)}$ is defined as

$$H(P,C) := \begin{bmatrix} (E_n + PC)^{-1} & -P(E_m + CP)^{-1} \\ C(E_n + PC)^{-1} & (E_m + CP)^{-1} \end{bmatrix} \tag{1}$$

provided that $\det(E_n + PC)$ is a nonzerodivisor of $\mathcal{A}$. This $H(P,C)$ is the transfer matrix from $[\, u_1^t \quad u_2^t \,]^t$ to $[\, e_1^t \quad e_2^t \,]^t$ of the feedback system Σ. If (i) $\det(E_n + PC)$ is a nonzerodivisor of $\mathcal{A}$ and (ii) $H(P,C) \in \mathcal{A}^{(m+n)\times(m+n)}$, then we say that the plant P is *stabilizable*, P is *stabilized* by C, and C is a *stabilizing controller* of P. In the definition above, we do not mention the causality of the stabilizing controller. However, it is known that if a causal plant is stabilizable, there exists a causal stabilizing controller of the plant[9].

Another matrix $W(P,C) \in (\mathcal{F})_{m+n}$ is defined as

$$W(P,C) := \begin{bmatrix} C(E_n + PC)^{-1} & -CP(E_m + CP)^{-1} \\ PC(E_n + PC)^{-1} & P(E_m + CP)^{-1} \end{bmatrix} \tag{2}$$

provided that $\det(E_n + PC)$ is a nonzerodivisor of $\mathcal{A}$. This $W(P,C)$ is the transfer matrix from $[\, u_1^t \quad u_2^t \,]^t$ to $[\, y_1^t \quad y_2^t \,]^t$ of the feedback system Σ. It is well known that $H(P,C)$ is over $\mathcal{A}$ if and only if $W(P,C)$ is over $\mathcal{A}$.

3 Previous Result

The parameterization of stabilizing controllers that *does not require the coprime factorizability* of the plant was originally given in [7]. Here we briefly outline this parameterization. Let $\mathcal{H}$ be the set of $H(P,C)$'s with all stabilizing controllers C. This set $\mathcal{H}$ and all stabilizing controllers are obtained as in the following way. Let H_0 be $H(P,C_0)$, where C_0 is a stabilizing controller of P.

Let $\Omega(Q)$ be a matrix defined as follows:

$$\Omega(Q) := (H_0 - \begin{bmatrix} E_n & O_{n\times m} \\ O_{m\times n} & O_{m\times m} \end{bmatrix})Q(H_0 - \begin{bmatrix} O_{n\times n} & O_{n\times m} \\ O_{m\times n} & E_m \end{bmatrix}) + H_0 \tag{3}$$

with a stable causal and square matrix Q of size $m+n$. Then we have the identity

$$\mathcal{H} = \{\Omega(Q) \,|\, Q \text{ is stable causal and } \Omega(Q) \text{ is nonsingular}\} \tag{4}$$

(Theorems 4.2 and 4.3 of [7]). Hence any stabilizing controller has the following form:

$$-\left[\, O_{m\times n} \quad E_m \,\right] \Omega(Q)^{-1} \begin{bmatrix} E_n \\ O_{m\times n} \end{bmatrix}, \tag{5}$$

provided that $\Omega(Q)$ is nonsingular.

The parameterization above is given by parameter matrix Q without the coprime factorizability of the plant. Nevertheless, this needs $(m+n)^2$ parameters even if the plant has a doubly coprime factorization. Recall that if we can use Youla parameterization, then the size of parameter matrix is mn. This means that there may exist unnecessary parameters in the parameterization above. The result of this paper will be to eliminate such unnecessary parameters.

4 Doubly Coprime Factorizability of the Stabilized Closed Feedback System

We state here the key results of the new parameterization. Recall first the following result[9].

Proposition 4.1 (Proposition 4.1 of [9]) *Suppose that P and C are matrices over $\mathcal{F}$. Suppose further that $\det(E_n + PC)$ is a unit of $\mathcal{F}$. Then $\mathcal{T}_{H(P,C),\mathcal{A}} \simeq \mathcal{T}_{P,\mathcal{A}} \oplus \mathcal{T}_{C,\mathcal{A}}$ holds.*

If C is a stabilizing controller of P, then the matrix $H(P,C)$ is over $\mathcal{A}$, so that $\mathcal{T}_{H(P,C),\mathcal{A}}$ is free. Thus by Proposition 4.1, $\mathcal{T}_{P,\mathcal{A}} \oplus \mathcal{T}_{C,\mathcal{A}}$ is also free. This leads that the plant $\mathrm{Diag}(C,P)$ has a right-coprime factorization over $\mathcal{A}$. The analogous statement holds for left-coprime factorization as well. From these observations, we give the following proposition.

Proposition 4.2 *Suppose that $C_0 \in \mathcal{F}^{m\times n}$ is a stabilizing controller of the plant $P \in \mathcal{P}^{n\times m}$. Then $P_1 := \mathrm{Diag}(C_0, P)$ has both right- and left-coprime factorizations over $\mathcal{A}$.*

Proof. Let

$$
\begin{aligned}
N_1 &= \widetilde{N}_1 = \begin{bmatrix} C_0(E_n + PC_0)^{-1} & -C_0P(E_m + C_0P)^{-1} \\ PC_0(E_n + PC_0)^{-1} & P(E_m + C_0P)^{-1} \end{bmatrix}, \\
D_1 &= \begin{bmatrix} (E_n + PC_0)^{-1} & -P(E_m + C_0P)^{-1} \\ C_0(E_n + PC_0)^{-1} & (E_m + C_0P)^{-1} \end{bmatrix}, \quad Y_1 = \widetilde{Y}_1 = \begin{bmatrix} O & E_n \\ -E_m & O \end{bmatrix}, \\
\widetilde{D}_1 &= \begin{bmatrix} (E_m + C_0P)^{-1} & -C_0(E_n + PC_0)^{-1} \\ P(E_m + C_0P)^{-1} & (E_n + PC_0)^{-1} \end{bmatrix}, \quad X_1 = \widetilde{X}_1 = \begin{bmatrix} E_n & O \\ O & E_m \end{bmatrix}.
\end{aligned}
$$

Note here that the matrices above are over $\mathcal{A}$. Using them, we have $P_1 = N_1D_1^{-1} = \widetilde{D}_1^{-1}\widetilde{N}_1$, $\widetilde{Y}_1N_1 + \widetilde{X}_1D_1 = E_{m+n}$, $\widetilde{N}_1Y_1 + \widetilde{D}_1X_1 = E_{m+n}$. Hence (N_1, D_1) and $(\widetilde{D}_1, \widetilde{N}_1)$ are right- and left-coprime factorizations of P_1, respectively. □

As a derivative of Proposition 4.2, we immediately have following proposition, which becomes the key idea of the new parameterization.

Proposition 4.3 *A plant $P \in \mathcal{P}^{n\times m}$ is stabilizable if and only if there exists a transfer function $F \in \mathcal{F}^{m'\times n'}$ with $0 \le m' \le m$ and $0 \le n' \le n$ such that plant $\mathrm{Diag}(F, P)$ has both right- and left-coprime factorizations over $\mathcal{A}$. (If $m' = n' = 0$, $\mathrm{Diag}(F, P)$ is considered as P.)*

A remarkable feature of the proposition above is that if a plant is stabilizable, in order to make a block diagonal with the plant, the size of the transfer matrix we need is *at most* m by *at most* n.

5 New Parameterization of Stabilizing Controllers

Now we give a parameterization of stabilizing controllers.

Let P be a stabilizable causal plant of $\mathcal{P}^{n\times m}$. Further, as in Proposition 4.3, let $F \in \mathcal{F}^{m'\times n'}$ with $m' \le m$ and $n' \le n$ be a transfer matrix such that the plant $P_1 := \mathrm{Diag}(F, P)$ has both right- and left-coprime factorizations over $\mathcal{A}$ ((a) of Figure 2). From the stabilizing controllers of the plant $\mathrm{Diag}(P, F)$, we will obtain all the stabilizing controllers of the original plant P.

Suppose that $P_1 = N_1D_1^{-1} = \widetilde{N}_1^{-1}\widetilde{D}_1 \in \mathcal{F}^{(m'+n)\times(m+n')}$, where

$$
\widetilde{Y}_1N_1 + \widetilde{X}_1D_1 = E_{m+n'}, \quad \widetilde{N}_1Y_1 + \widetilde{D}_1X_1 = E_{m'+n},
$$

and N_1, D_1, $\widetilde{N}_1$, $\widetilde{D}_1$, Y_1, X_1, $\widetilde{Y}_1$, $\widetilde{X}_1$ are matrices over $\mathcal{A}$. Suppose further that C_{10} is a stabilizing controller of P_1 such that $C_{10} = \widetilde{X}_1^{-1}\widetilde{Y}_1 = Y_1X_1^{-1}$.

Then all stabilizing controllers of the plant P_1 is parameterized with parameter matrices $R, S \in$

$\mathcal{A}^{(m+n')\times(m'+n)}$ as follows[6]:

$$(\widetilde{X}_1 - R\widetilde{N}_1)^{-1}(\widetilde{Y}_1 + R\widetilde{D}_1), \tag{6}$$

$$(Y_1 + N_1 S)(X_1 - D_1 S)^{-1}. \tag{7}$$

Let C_1 be an arbitrary stabilizing controller of P_1 with a parameter matrix $R \in \mathcal{A}^{(m+n')\times(m'+n)}$ as (6), that is, $C_1 = (\widetilde{X}_1 - R\widetilde{N}_1)^{-1}(\widetilde{Y}_1 + R\widetilde{D}_1)$. Then $W(P_1, C_1)$ is expressed as follows:

$$\begin{aligned} W(P_1, C_1) &= \begin{bmatrix} C_1(E_{m'+n} + P_1C_1)^{-1} & -C_1P_1(E_{m+n'} + C_1P_1)^{-1} \\ P_1C_1(E_{m'+n} + P_1C_1)^{-1} & P_1(E_{m+n'} + C_1P_1)^{-1} \end{bmatrix} \\ &= \begin{bmatrix} D_1(\widetilde{Y}_1 + R\widetilde{D}_1) & D_1(\widetilde{X}_1 - R\widetilde{N}_1) - E_{m+n'} \\ N_1(\widetilde{Y}_1 + R\widetilde{D}_1) & N_1(\widetilde{X}_1 - R\widetilde{N}_1) \end{bmatrix}. \end{aligned}$$

For a moment, let us proceed graphically with the block diagrams in Figure 2. The block diagram of the feedback system consisting of P_1 and C_1 is as in (b) of Figure 2. Decompose the stabilizing controller C_1 as follows:

$$\begin{bmatrix} C_{111} & C_{112} \\ C_{121} & C_{122} \end{bmatrix} := C_1,$$

where $C_{111} \in \mathcal{F}^{n'\times m'}$, $C_{112} \in \mathcal{F}^{n'\times n}$, $C_{121} \in \mathcal{F}^{m\times m'}$, $C_{122} \in \mathcal{F}^{m\times n}$. Consider now the following matrix

$$\begin{bmatrix} O_{m\times n'} & E_m & O_{m\times m'} & O_{m\times n} \\ O_{n\times n'} & O_{n\times m} & O_{n\times m'} & E_n \end{bmatrix} W(P_1, C_1) \begin{bmatrix} O_{m'\times n} & O_{m'\times m} \\ E_n & O_{n\times m} \\ O_{n'\times n} & O_{n'\times m} \\ O_{m\times n} & E_m \end{bmatrix}. \tag{8}$$

Note that this matrix is the transfer matrix from $[\, u_{12} \;\; u_{22} \,]^t$ to $[\, y_{12} \;\; y_{22} \,]^t$ in (c) of Figure 2. Let

$$C_{\text{New}} = C_{122} - C_{121}(E_{m'} + FC_{111})^{-1}FC_{112} \in \mathcal{F}^{m\times n}. \tag{9}$$

Using this C_{New}, (c) of Figure 2 can be rewritten as (d) of the figure. This is a feedback system of P and C_{New}. Hence C_{New} is a new stabilizing controller of the plant P. One can check straightforwardly but tediously that the matrix of (8) is equal to $W(P, C_{\text{New}})$, that is,

$$W(P, C_{\text{New}}) = \begin{bmatrix} O_{m\times n'} & E_m & O_{m\times m'} & O_{m\times n} \\ O_{n\times n'} & O_{n\times m} & O_{n\times m'} & E_n \end{bmatrix} W(P_1, C_1) \begin{bmatrix} O_{m'\times n} & O_{m'\times m} \\ E_n & O_{n\times m} \\ O_{n'\times n} & O_{n'\times m} \\ O_{m\times n} & E_m \end{bmatrix}. \tag{10}$$

Having the observation above, we are now ready to present the main result of this paper. To state it, we provide some notations. Let $\mathcal{W}(P;\mathcal{R})$ denote the set of $W(P,C)$'s such that C is an $\mathcal{R}$-stabilizing controller of P and $\mathcal{S}(P;\mathcal{R})$ the set of all $\mathcal{R}$-stabilizing controllers. Then the set $\mathcal{W}(P;\mathcal{R})$ is given as $\{W(P,C)\,|\,C\in\mathcal{S}(P;\mathcal{R})\}$. Conversely, once we obtain $\mathcal{W}(P;\mathcal{R})$, it is also easy to obtain the set $\mathcal{S}(P;\mathcal{R})$ as follows:

$$\mathcal{S}(P;\mathcal{R}) \;=\; \{W_{11}(E_n-W_{21})^{-1} \;|\; \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22}\end{bmatrix}\in\mathcal{W}(P;\mathcal{R}),\; E_n-W_{21} \text{ is nonsingular}\},$$

where $W_{11}\in\mathcal{R}^{m\times n}$, $W_{12}\in(\mathcal{R})_m$, $W_{21}\in(\mathcal{R})_n$, $W_{22}\in\mathcal{R}^{n\times m}$ (subject to $P\in\mathcal{F}(\mathcal{R})^{n\times m}$). This implies that obtaining $\mathcal{S}(P;\mathcal{A})$ and obtaining $\mathcal{W}(P;\mathcal{A})$ are equivalent to each other.

Now we can state the main result of this paper.

Theorem 5.1 *Suppose that $P\in\mathcal{P}^{n\times m}$ is a stabilizable plant and that $F\in\mathcal{F}^{m'\times n'}$ with $m'\le m$ and $n'\le n$ is a transfer matrix such that $P_1 := \mathrm{Diag}(F,P)$ has both right- and left-coprime factorizations over $\mathcal{A}$. Let (N_1,D_1) and $(\widetilde{N}_1,\widetilde{D}_1)$ be right- and left-coprime factorizations of P_1 over $\mathcal{A}$, respectively. Further let*

$$\begin{aligned}\Psi(R) \;=\; & \begin{bmatrix} O_{m\times n'} & E_m & O_{m\times m'} & O_{m\times n} \\ O_{n\times n'} & O_{n\times m} & O_{n\times m'} & E_n\end{bmatrix} \\ & \times\begin{bmatrix} D_1(\widetilde{Y}_1+R\widetilde{D}_1) & D_1(\widetilde{X}_1-R\widetilde{N}_1)-E_{m+n'} \\ N_1(\widetilde{Y}_1+R\widetilde{D}_1) & N_1(\widetilde{X}_1-R\widetilde{N}_1)\end{bmatrix}\begin{bmatrix} O_{m'\times n} & O_{m'\times m} \\ E_n & O_{n\times m} \\ O_{n'\times n} & O_{n'\times m} \\ O_{m\times n} & E_m\end{bmatrix}.\end{aligned} \tag{11}$$

Then

$$\mathcal{W}(P;\mathcal{A}) = \{\Psi(R)\;|\; R\in\mathcal{A}^{(m+n')\times(m'+n)},\; E_{m'+n}-N_1(\widetilde{Y}_1+R\widetilde{D}_1) \text{ is nonsingular }\}. \tag{12}$$

The matrix function $\Psi(\cdot)$ of (11) is similar with $\Omega(\cdot)$ of [7].

Proof. The existence of the matrix F is known by Proposition 4.3. We prove this proposition by showing "$\supset$" and "$\subset$" of (12).

"$\supset$". For any stabilizing controller C_1 of P_1, one can construct a stabilizing controller C_{New} of P such that (10) holds, as in Figure 2. Thus, for any parameter matrix R such that $E_{m'+n}-N_1(\widetilde{Y}_1+R\widetilde{D}_1)$ is nonsingular, there exists a stabilizing controller C_{New} of P such that $\Psi(R)=W(P,C_{\mathrm{New}})$.

"$\subset$". Let C_0 be a stabilizing controller of P. By Theorem 3.3 of [9], F is stabilizable. Let C_F be a stabilizing controller of F. Then obviously $\mathrm{Diag}(C_F,C_0)$ is a stabilizing controller of $\mathrm{Diag}(F,P)$.

Hence there exists an R such that

$$W(P_1, \mathrm{Diag}(C_F, C_0)) = \begin{bmatrix} D_1(\widetilde{Y}_1 + R\widetilde{D}_1) & D_1(\widetilde{X}_1 - R\widetilde{N}_1) - E_{m+n'} \\ N_1(\widetilde{Y}_1 + R\widetilde{D}_1) & N_1(\widetilde{X}_1 - R\widetilde{N}_1) \end{bmatrix}$$

In this case, $\Psi(R) = W(P, C_0)$ holds. □

We remark that the number of parameters of the parameter matrix R is $(m+n') \times (m'+n)$. Since $n' \leq n$ and $m' \leq m$ hold, the number of parameters is decreased by comparing with the previous result in Section 3.

In addition to Theorem 5.1, the author has the following conjecture, which is not proved yet, as its derivative.

Conjecture 5.1 *Let P, m', n' N_1, D_1, $\widetilde{N}_1$, $\widetilde{D}_1$ be the same as Theorem 5.1. Suppose that C_0 is a stabilizing controller of P. Then*

$$\begin{aligned} &\mathcal{W}(P;\mathcal{A}) \qquad (13) \\ &= \left\{ \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} := W(P, C_0) + \begin{bmatrix} O_{m\times n'} & E_m & O_{m\times m'} & O_{m\times n} \\ O_{n\times n'} & O_{n\times m} & O_{n\times m'} & E_n \end{bmatrix} \begin{bmatrix} D_1 \\ N_1 \end{bmatrix} R \begin{bmatrix} \widetilde{D}_1 & -\widetilde{N}_1 \end{bmatrix} \right. \\ &\qquad \left. \times \begin{bmatrix} O_{m'\times n} & O_{m'\times m} \\ E_n & O_{n\times m} \\ O_{n'\times n} & O_{n'\times m} \\ O_{m\times n} & E_m \end{bmatrix} \mid R \in \mathcal{A}^{(m+n')\times(m'+n)},\ E_n - W_{21} \text{ is nonsingular} \right\} \end{aligned}$$

($W_{11} \in \mathcal{A}^{m\times n}$, $W_{12} \in (\mathcal{A})_m$, $W_{21} \in (\mathcal{A})_n$, $W_{22} \in \mathcal{A}^{n\times m}$).

6 Relationship with the previous results

The result of the previous section can reduce to the previous results, that is, Youla parameterization and the result given in [7].

Youla parameterization

Suppose that plant P has both right- and left-coprime factorizations. Then n' and m' in Proposition 4.3 are zero. Then it is easy to see that the right hand side of (13) is just a Youla parameterization.

Result of [7]

Suppose that the plant $P \in \mathcal{P}^{n\times m}$ is stabilizable and C_0 a stabilizing controller of P. By Proposition 4.2, this C_0 can be F of Proposition 4.3. Let us consider that $F := C_0$. Then the right hand side of (13) is expressed as

$$W(P,C_0) + (W(P,C_0) + \begin{bmatrix} O & E_m \\ O & O \end{bmatrix})R(-W(P,C_0) + \begin{bmatrix} O & O \\ E_n & O \end{bmatrix}).$$

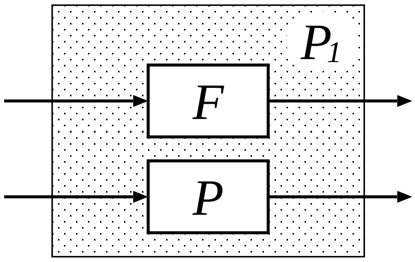
Let $W(P,C)$ be the matrix expression above. Then $H(P,C)$ is obtained as follows from $W(P,C)$ (cf. [14, p.102]):

$$\begin{aligned} H(P,C) &= E_{m+n} - \begin{bmatrix} O & E_n \\ -E_m & O \end{bmatrix} W(P,C) \qquad (14) \\ &= (H(P,C_0) - \begin{bmatrix} E_n & O_{n\times m} \\ O_{m\times n} & O_{m\times m} \end{bmatrix})Q(H(P,C_0) - \begin{bmatrix} O_{n\times n} & O_{n\times m} \\ O_{m\times n} & E_m \end{bmatrix}) + H(P,C_0) \end{aligned}$$

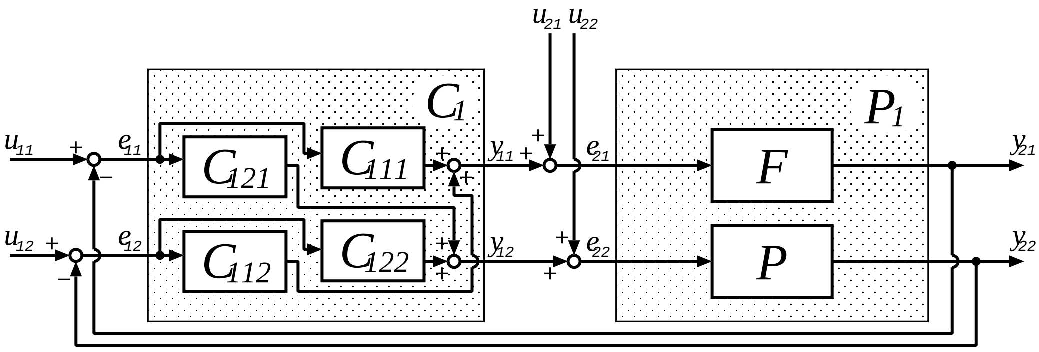
(14) is same as (3).

7 Related Works

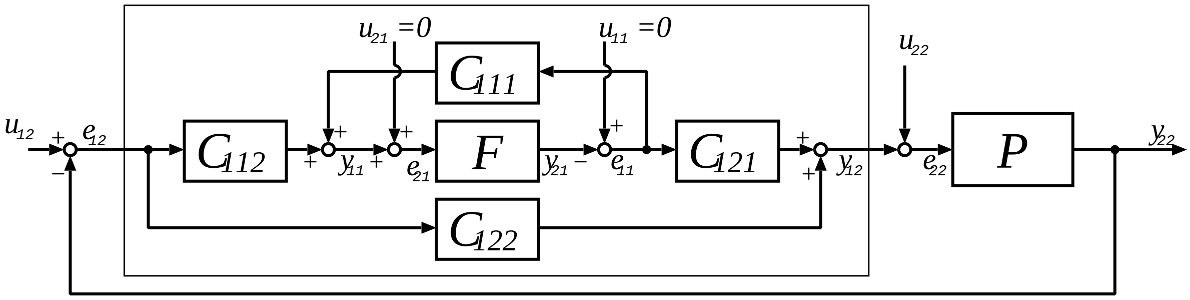
Recently the author in [8] has given the parameterization method of all stabilizing controllers which requires *only one* of right-/left-coprime factorizations. The relationship between the results of this paper and [8] should be investigated. The author considers that Proposition 4.3 may have some relation to the result of [8].



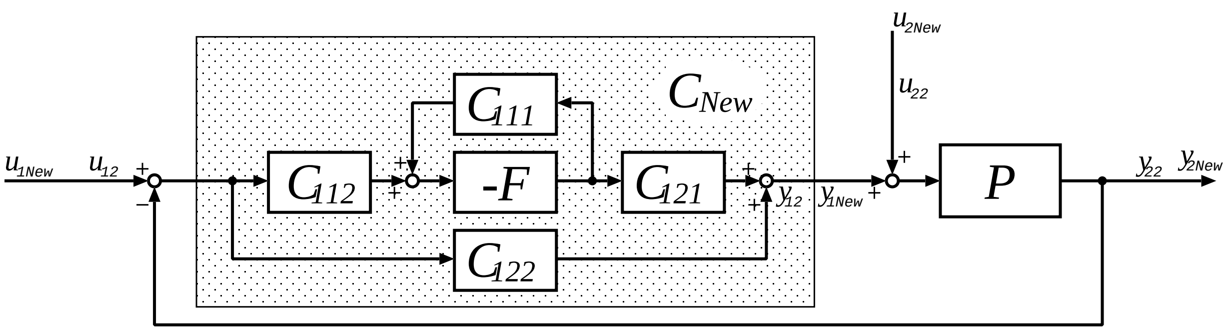
(a) New plant $P_1 := \mathrm{Diag}(F, P)$.



(b) New plant P_1 and its stabilizing controller C_1.



(c) Relocating the components of P_1 and C_1.



(d) Original plant P and its newly obtained stabilizing controller C_{New}.

Figure 2: Construction of a stabilizing controller of the plant.

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