Application of Nonlinear Lyapunov-Based Controllers and Observers To Gasoline Direct Injection Engine Charge and Torque Control

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Abstract

The paper discusses several Lyapunov-based controller and observer techniques that enhance the conventional Speed-Gradient control design approach. Our developments cover the treatment of the derivative action, time-delays, backstepping via dynamic surface control, and feedforward governor to satisfy pointwise-in-time constraints. Some of these enhancements have been already proposed in the prior literature in the context of specific automotive applications but are generalized here; other developments such as a feedforward governor are new. We also review several concrete applications of these techniques to control of gasoline direct injection engines, including torque, air-to-fuel ratio and charge control.

1 Introduction

This paper reviews several enhancements to the Speed-Gradient (SG) control design approach [4, 5] which is a Lyapunov design technique where a selected performance (or goal) function Q is dynamically minimized with a choice of control input. Formally, SG approach falls within the class of Control Lyapunov Function (CLF)-based methods wherein Q is a CLF [14]; unlike in other CLF methods, Q is selected a priori to reflect and appropriately weight different performance objectives while closed loop stability is verified a posteriori. The SG design approach can also be intuitively interpreted as a form of one-step ahead Model Predictive Control. Traditionally, Speed-Gradient technique yields either proportional (SG-P) or proportional plus integral (SG-PI) nonlinear controllers.

In this paper, we discuss several enhancements to the conventional SG approach, including the treatment of derivative action (so that SG-PD controllers are obtained), time-delays, backstepping via dynamic surface control, and feedforward governor to satisfy pointwise-in-time constraints. Most of these enhancements (excluding feedforward governor) have been already proposed in the treatment of special automotive applications in the literature, including our own work. In this paper we review these ideas and extend them to more general classes of systems and problems. We also review several applications to automotive engine control where these techniques proved useful.

The SG-PD controllers are obtained via integration of the original SG-P controller with an input observer for the unknown input additive to the control signal. This observer basically estimates a derivative of a certain system input and the resulting SG-PD controller can reject additive disturbances with bounded rate of change. Assuming that there are no control constraints, unbounded disturbances are allowed.

Backstepping via dynamic surface control requires that the integral state of the PI controller be limited (via artificial saturation) and that the integral gain be sufficiently large. Our special result on systems with time-delays basically requires that the proportional feedback from the control not affected by the delay be sufficiently strong. The feedforward governor is a modification of a concept of a reference governor [1, 6] but in our case the context is a stabilization problem not a tracking problem and the governor adjusts the feedforward value and not the reference command value.

The paper is organized as follows. We review the Speed-Gradient (SG) approach in Section 2. We then treat various design enhancements: SG-PD controllers in Section 3, backstepping via dynamic surface control in Section 4, application to time-delayed in Section 5, and feedforward governors in Section 6. Several of these ideas have been motivated by prior applications in the area of gasoline direct injection (GDI) engine control, see [9, 10, 11, 13, 16, 18]. Section 7 will review specific applications of the design techniques to air-to-fuel ratio and torque regulation and to charge control in the GDI engines.

2 Speed-Gradient Approach

Consider a nonlinear control system of the form

$$\dot{x} = f(x) + g(x)u, \tag{2.1}$$

where x is an n-vector state and u is an m-vector control. Given the desired equilibrium of (2.1), x_d, suppose there is a feedforward control u_d that can be used to maintain this equilibrium, i.e.,

$$f(x_d) + g(x_d)u_d = 0.$$

Suppose now the input u is generated by applying, in addition to the above feed-forward control term, u_d, a proportional feedback term, u_p, and an integral feedback term, θ. Furthermore, suppose the effect of uncertainties and parameter variations can be represented by a constant disturbance w additive to the control input. Thus

$$u = u_d + u_p + \theta + w. \tag{2.2}$$

The feedback components u_p and θ are determined with the help of Speed-Gradient (SG) approach [4, 5].

To be specific, consider a derivation of the SG controller motivated by one step ahead model predictive control (MPC) considerations. Let $Q(x)$ be the so called *goal* or *performance* function. It satisfies $Q(x_d) = 0$, $Q(x) \geq 0$, and is chosen so that the requirement

$$Q(x(t)) \to 0 \quad \text{as } t \to \infty \tag{2.3}$$

captures the control design objectives. For example, one possible choice for Q is

$$Q = \sum_{i=1}^{n} \gamma_i (x_i - x_{d,i})^2,$$

where the weights γ_i reflect the relative priority of tracking in different state channels. Assuming that $u(t)$ is constant on $[t, t+\Delta t[$, $Q(x(t+\Delta t))$ can be approximated for small Δt as

$$\begin{aligned} &Q(x(t+\Delta t)) \approx Q(x(t)) + \dot{Q}(t)\Delta t \\ &= Q(x(t)) + \Big(L_f Q(x(t)) + L_g Q(x(t)) u(t) \Big) \Delta t, \end{aligned} \tag{2.4}$$

where,

$$L_f Q(x) \triangleq \frac{\partial Q}{\partial x}(x) f(x), \quad L_g Q(x) \triangleq \frac{\partial Q}{\partial x}(x) g(x).$$

The approximate cost (2.4) can be *regularized* by augmenting a penalty on the proportional correction and on the incremental change of the integral correction:

$$\begin{aligned} &Q(x(t)) + \Big(L_f Q(x(t)) + L_g Q(x(t)) \cdot (u_d + u_p(t) + \theta(t) + w) \Big) \cdot \Delta t + \\ &\frac{1}{2} u_p^{\mathrm{T}}(t) \left(\frac{\Pi}{\Delta t} \right)^{-1} u_p(t) + \frac{1}{2} (\theta(t) - \theta(t - \Delta))^{\mathrm{T}} \Gamma^{-1} (\theta(t) - \theta(t - \Delta t)) \\ &\to \min, \end{aligned} \tag{2.5}$$

where $\Pi > 0$, $\Gamma > 0$. The direct minimization of the cost (2.5) with respect to $u_p(t)$ and $\theta(t)$ yields

$$u_p = -\Pi (L_g Q)^{\mathrm{T}}, \tag{2.6}$$

and

$$\theta(t) = \theta(t - \Delta t) - \Gamma (L_g Q)^{\mathrm{T}} \Delta t.$$

The last equation is a discrete-time version of the continuous-time update law

$$\dot{\theta} = -\Gamma (L_g Q)^{\mathrm{T}}. \tag{2.7}$$

The term $(L_g Q)^{\mathrm{T}} = (\frac{\partial Q}{\partial x} g)^{\mathrm{T}}$ that appears on the right-hand of (2.6), (2.7) can also be computed as $\nabla_u \dot{Q}$ which is the gradient of the speed of change of Q. Thus

$$u_p = -\Pi \nabla_u \dot{Q}, \ \dot{\theta} = -\Gamma \nabla_u \dot{Q}.$$

Hence the control design (2.2),(2.6),(2.7) is referred to as "Speed-Gradient." It can also be interpreted as a version of nonlinear PI controller.

The stability analysis for the closed loop system *typically* proceeds by considering a Lyapunov function candidate of the form

$$V = Q + \frac{1}{2} (\theta + w)^{\mathrm{T}} \Gamma^{-1} (\theta + w). \tag{2.8}$$

Its time derivative along the trajectories of the system (2.1), (2.2), (2.6), (2.7), under an additional assumption that $\dot{w} \equiv 0$, satisfies

$$\dot{V} = (L_f Q + L_g Q u_d) - (L_g Q)\Pi(L_g Q)^{\mathrm{T}}.$$

The following sufficient stability conditions follows by application of the Barbalat's lemma:

Proposition: Suppose that (i) $Q(x_d) = 0$, $Q(x) > 0$ if $x \neq x_d$ and Q is twice continuously differentiable in $\Upsilon_c = \{x : \ Q(x) \leq c\}$; (ii) the achievability condition [5] holds, i.e.,

$$-\rho(Q(x)) + L_g Q(x)\Lambda(L_g Q(x))^{\mathrm{T}} \geq (L_f Q(x) + L_g Q(x) u_d), \tag{2.9}$$

for some $m \times m$ matrix $\Lambda \geq 0$, a continuously differentiable function ρ that satisfies

$$\rho(0) = 0, \quad \rho(z) > 0 \ \text{ if } z > 0,$$

and for all $x \in \Upsilon_c$; (iii) f, g are twice continuously differentiable and bounded together with the first and second derivatives in Υ_c; (iv) the initial conditions $(x(0), \theta(0))$ satisfy

$$Q(x(0)) + \frac{1}{2}(\theta(0) + w)^{\mathrm{T}}\Gamma^{-1}(\theta(0) + w) \leq c; \tag{2.10}$$

(v) $\Pi > \Lambda$, $\Gamma > 0$; (vi) $\dot{w} \equiv 0$. Then the closed loop system is Lyapunov stable and the closed loop trajectories satisfy

$$\lim_{t\to\infty} Q(x(t)) = 0, \quad \lim_{t\to\infty} x(t) = x_d,$$

$$Q(x(t)) \leq c \ \ \forall t.$$

If, furthermore, the matrix $g(x_d)$ has a full column rank, then

$$\lim_{t\to\infty} \theta(t) = -w.$$

The achievability condition (2.9) may be checked either analytically or numerically. The purely numerical approach proceeds by setting $\Lambda = 0$, and maximizing $(L_f Q(x) + L_g Q(x) u_d)$ subject to $\tilde{c} \leq Q(x) \leq c$, where $\tilde{c} > 0$. Let $z(\tilde{c})$ denote the maximum. By analyzing $z(\tilde{c})$ as a function of $\tilde{c}$, a continuously differentiable function ρ, $\rho(0) = 0$, $\rho(\tilde{c}) > 0$ if $\tilde{c} > 0$, that yields $\tilde{z}(\tilde{c}) \leq -\rho(\tilde{c})$ may be prescribed. If such ρ can be found, $\Lambda = 0$, ρ satisfy the inequality in (2.9) for $x \in \Upsilon_c$. If no such ρ can be found, $\Lambda > 0$ needs to be considered. As a heuristic approach, tuning the controller to work well in simulations with a $\Pi > 0$ and then trying Λ slightly less than Π works quite well.

In a situation when $Q(x) \geq 0$ but $Q(x) > 0$ for all $x \neq x_d$ does not hold, $x(t)$ may not, in general, converge to x_d. It is still possible, however, to demonstrate that $L_f Q(x(t)) \to 0$ and $L_g Q(x(t)) \to 0$. Note that $L_f Q(x) = 0$, $L_g Q(x) = 0$ is a system of $(m+1)$ equations in n unknowns. When $(m+1) \geq n$, this system may, frequently, have x_d as the unique solution, and, hence, $x(t) \to x_d$. If $n = m$ and the $m \times m$ matrix $g(x_d)$ is nonsingular, $\theta(t) \to -w$.

If $n > (m+1)$, it is necessary to demonstrate that the closed loop trajectories are bounded and that $L_f Q(x(t)) = 0$, $L_g Q(x(t)) = 0$, $\dot{x}(t) = f(x(t)) + g(x(t))u_d$, for all t, imply that $x(t) \equiv x_d$. This can be checked through an observability-like condition in [15].

Note that the form of the feedback (2.6) suggests the connection with the conventional Control Lyapunov Function methods, in particular, $L_g V$-techniques [14]. The difference is mainly in the approach: Q is selected by the designer to capture the transient performance objectives in the SG approach; Q is constructed as a Control Lyapunov Function in the methodologies covered in [14]. The strength of SG approach is in the strong linkage between control objectives and the selection of the function Q. This greatly helps in the tuning process. Specifically, if we are not satisfied with the transient response, we adjust the weights in Q or augment barrier functions. The weakness of SG approach is that the procedures to modify Q are not readily available if the achievability condition does not hold. The achievability condition, on the other hand, is only sufficient and the stability may be verified by other procedures. For example, numerical simulations followed up by the numerical construction of a Lyapunov function can be used.

3 SG-PD Controllers

SG-PD controllers use an unknown input observer (of derivative type) to estimate unmeasured exogenous disturbance inputs or a model mismatch. If actuators are sufficiently fast, they may be used to counteract the disturbance or the effect of uncertainty.

Specifically, consider a nonlinear control system (2.1) under the nominal feedforward, u_d, and SG-P feedback law, $u_p = -\Pi (L_g Q)^{\mathrm{T}}$:

$$\dot{x} = \tilde{f}(x) + g(x)(\tilde{u} + w), \quad \tilde{f}(x) \stackrel{\Delta}{=} f(x) + g(x)(u_d + u_p), \tag{3.11}$$

and where the auxiliary control $\tilde{u}$ is to be determined and where the disturbance vector w may represent, for example, modeling errors, errors in the feedforward control, or actual unknown inputs acting on the system. Unlike the case in Section 2 (see (2.2)) , here w may be time-varying.

One approach to compensating for the effects of w is to first estimate w from the state measurements and then compensate for w in the control law, i.e.,

$$\tilde{u} = -\hat{w},$$

where $\hat{w}$ is an estimate of w. To generate $\hat{w}$ (an m-dimensional vector) suppose z is an m-dimensional output of the system, which can be measured,

$$z = h(x).$$

Then,

$$\dot{z} = L_{\tilde{f}} h + L_g h(\tilde{u} + w).$$

Thus one arrives at the following input estimation problem:

$$\dot{z} = y + v,$$

where

$$y = L_{\tilde{f}}h + L_g h \cdot \tilde{u}, \quad v = L_g h \cdot w,$$

and where v needs to be estimated. An observer for v can be defined by

$$\begin{aligned}
\hat{v} &= \gamma z - \varepsilon, \quad \gamma > 0, \\
\dot{\varepsilon} &= \gamma \cdot (\hat{v} + y). \\
\hat{w} &= (L_g h)^{-1} \hat{v}.
\end{aligned}$$

Here $\hat{v}$, ε are vectors of the same dimension as z and w. For this estimation scheme to be well-defined, the inverse of the square matrix $(L_g h)(x)$ must exist and be uniformly bounded for all x. The observer is similar to filters that estimate a signal derivative.

Technical conditions can be given guaranteeing that if $\sup_t |\dot{w}(t)| < \infty$ then by selecting $\gamma > 0$ sufficiently large, $x(t)$ can be rendered ultimately bounded in any desired neighborhood of x_d. Stronger than needed sufficient technical conditions are, for example: (i) $(L_g h)(x)$ is a constant invertible matrix; (ii) a twice continuously differentiable performance function Q for the nominal SG-P design satisfies $Q(x_d) = 0$, $Q(x) > 0$ for $x \neq x_d$; (iii) $\sup_x \frac{||L_g Q(x)||^2}{Q(x)} \leq c$, $\infty > c > 0$; (iv) $L_{\tilde{f}} Q(x) \leq -\rho \cdot Q$, $\rho > 0$. These conditions if hold locally guarantee locally (and if hold globally guarantee globally) that an inequality of the form

$$\begin{aligned}
\dot{V} &\leq -\rho \cdot Q + L_g Q (L_g h)^{-1} (v - \hat{v}) - \gamma (v - \hat{v})^2 + (\hat{v} - v)\dot{\hat{v}} \\
&\leq -\gamma_1 V + \gamma_2
\end{aligned}$$

holds, where $V = Q + \frac{1}{2}(v - \hat{v})^2$, and γ_2/γ_1 can be made as small as desired by selecting γ sufficiently large.

Note that if $|w(t)|$ is bounded, it may be also possible to suppress it in (3.11) by a high gain feedback. The disturbance compensation as opposed to disturbance suppression has several advantages. Specifically, in our unknown input observer-based scheme it is only required that the observer gain be sufficiently large and that $|\dot{w}(t)|$ be bounded (boundness of $|w(t)|$ is not required). Furthermore, if $|\dot{w}(t)| \to 0$ as $t \to \infty$ then $x(t) \to x_d$ as $t \to \infty$. This is a result resembling asymptotic properties of the integral control.

References [16, 17, 3] describe various automotive applications of this or a similar approach. In [17] the application is idle speed control and w models the load torque due to auxiliaries applied to engine cranshaft. In [3] the application is to vehicle speed control and w represents the uncertainty in the road grade. The terminology SG-PD control is probably introduced first in [3]. In [16] the application is to engine partial pressure of air control and w represents an unknown EGR flow.

4 Backstepping Via Dynamic Surface Control

The development of SG-PI controller assumes that the control u appears explicitly in the expression for $\dot{Q}$. Backstepping via dynamic surface control [19] can be used in systems where the relative degree from u to Q is greater than one. Specifically, consider the case when the system is described by the following equations.

$$\begin{aligned}\dot{x} &= f(x) + g(x)(v + w), \\ \dot{v} &= u,\end{aligned} \tag{4.12}$$

where w is a constant unknown disturbance, u is the control input and the equation $\dot{v} = u$ represents (after appropriate transformations) the actuator dynamics.

Considering the system without the actuator dynamics, the SG-PI feedback law for v has the following form

$$\begin{aligned}v_0 &= v_d - \Pi(L_g Q)^{\mathrm{T}} + \theta, \\ \dot{\theta} &= -\Gamma(L_g Q)^{\mathrm{T}}.\end{aligned} \tag{4.13}$$

To extend this feedback law for v to the one for u, backstepping via the dynamic surface control can be applied:

$$\begin{aligned}\dot{v}_s &= -\tau v_s + \tau v_0 \\ u &= \alpha(v_s - v) + \dot{v}_s.\end{aligned} \tag{4.14}$$

The control u forces v to follow v_s, while v_s, in turn, is forced to follow v_0. The dynamic surface control represents a simplified backstepping procedure relying on dynamic, differentiation-like filters in place of potentially more complicated nonlinear transformations of the conventional backstepping procedure.

The properties of the closed loop system can be investigated using a Lyapunov function

$$V = Q(x) + \frac{1}{2}(v_s - v_0)^{\mathrm{T}}(v_s - v_0) + \frac{1}{2}(v - v_s)^{\mathrm{T}}(v - v_s) + \frac{1}{2}(\theta + w)^{\mathrm{T}}\Gamma^{-1}(\theta + w).$$

It can be verified that

$$\begin{aligned}\dot{V} &= L_f Q + L_g Q(v - v_s) + L_g Q(v_s - v_0) - (L_g Q)\Pi(L_g Q)^{\mathrm{T}} \\ &+ L_g Q v_d - \tau||v_s - v_0||^2 - \dot{v}_0(v_s - v_0) - \alpha||v_s - v||^2 \\ &\leq (L_f Q + L_g Q v_d - (LgQ)\Pi(L_g Q)^{\mathrm{T}}) + \frac{||L_g Q||^2}{2c_1} \\ &+ \frac{c_1||v - v_s||^2}{2} + \frac{||L_g Q||^2}{2c_2} + \frac{c_2||v_s - v_0||^2}{2} + \frac{||\dot{v}_d||}{2c_3} \\ &+ \frac{c_3||v_s - v_0||^2}{2} - \tau||v_s - v_0||^2 - \alpha||v_s - v||^2,\end{aligned}$$

for any positive c_1, c_2, c_3. Let $z = [x, v, v_s]^{\mathrm{T}}$, $V_z(z) = Q(x) + \frac{1}{2}||v_s - v_0||^2 + \frac{1}{2}||v - v_s||^2$. It follows that if $V_z(z) > c_0$ then $\dot{V} \leq 0$, where $c_0 = c_0(\tau, \alpha) \to 0$ as $\tau, \alpha \to \infty$. These inequalities guarantee ultimate boundness of $z(t)$ in an arbitrary small neighboorhood of the origin provided that the integral state of the controller θ is saturated to the interval $[-w_{max}, -w_{min}]$ (where w_{min} and w_{max} are minimum and maximum a priori known bounds on w, $w_{min} \leq w \leq w_{max}$) and the integral gain $\Gamma > 0$ is sufficiently large. Thus integrator state limiting and large integrator gain may be required when extending the SG-PI controller via dynamic surface control.

This observation is similar to the "bursting prevention" technique in adaptive control through the use of parameter projections and large adaptation gains [7]. A similar procedure and result for combining dynamic surface control with parameter adaptation has been reported in [2] in the context of a specific vehicle speed control problem.

5 Treatment of delays

Consider a nonlinear control system with delays in the state variables:

$$\dot{x} = f(x(t), x(t-\Delta)) + g(x(t), x(t-\Delta))u. \tag{5.15}$$

We assume that a performance function Q has been selected and the control (2.2)

$$u = u_d + u_p + \theta + w,$$

is defined by the SG method with

$$u_p = -\Pi(L_g Q)^{\mathrm{T}}, \quad \dot{\theta} = -\Gamma(L_g Q)^{\mathrm{T}}, \quad (L_g Q) = (\frac{\partial Q}{\partial x})g(x(t), x(t-\Delta)). \tag{5.16}$$

Consider a Lyapunov-like functional

$$V = \frac{1}{2}(\theta + w)^{\mathrm{T}}\Gamma^{-1}(\theta + w) + Q + \int_{t-\Delta}^{t} S(x(\tau))d\tau,$$

where $S(x)$ is to be determined. We obtain,

$$\begin{aligned}
\dot{V} &= \frac{\partial Q}{\partial x} f(x(t), x(t-\Delta)) \\
&+ \frac{\partial Q}{\partial x} g(x(t), x(t-\Delta))u_d + \frac{\partial Q}{\partial x} g(x(t), x(t-\Delta))u_p + S(x(t)) - S(x(t-\Delta)) \\
&= \frac{\partial Q}{\partial x} f(x(t), x(t)) + \frac{\partial Q}{\partial x} g(x(t), x(t))(u_d + u_p) \\
&+ \frac{\partial Q}{\partial x}\Big((f(x(t), x(t-\Delta)) - f(x(t), x(t))) + (g(x(t), x(t-\Delta)) - g(x(t), x(t))(u_d + u_p))\Big) \\
&+ S(x(t)) - S(x(t-\Delta)).
\end{aligned}$$

Suppose now the delay only affects the drift vector field f but not the control vector field g, i.e., $g(x(t), x(t-\Delta)) = g(x(t))$. Suppose further there exist positive functions $S(x) > 0$ and $R(x) > 0$ if $x \neq x_d$ such that

$$\frac{\partial Q}{\partial x}\Big(f(x(t), x(t)) + g(x(t))u_d - \Pi(\frac{\partial Q}{\partial x} g(x(t)))^{\mathrm{T}}\Big) + S(x(t)) + R(x(t)) < -\rho(Q), \tag{5.17}$$

and

$$R(x(t)) + S(x(t-\Delta)) \geq \frac{\partial Q}{\partial x}\Big(f(x(t), x(t-\Delta)) - f(x(t), x(t))\Big). \tag{5.18}$$

Then the closed loop stability follows from the theorems in [8]. The use of large proportional control gain Π can be essential in the applications to render (5.17) and (5.18) satisfied.

A somewhat different situation arises when there are two control inputs, with one acting through a non-delayed control vector field and another one acting through a delayed control vector field. Specifically, consider a system

$$\dot{x} = f(x(t), x(t-\Delta)) + g_1(x(t))u_1 + g_2(x(t-\Delta))u_2.$$

The nominal feedback

$$u_2 = -\Pi_2 (L_{g_2} Q)^{\mathrm{T}},$$

converts the term $g_2(x(t-\Delta))u_2$ into a drift term. A strong feedback from the control u_1 acting through a non-delayed vector field is thus essential to ensure stability. The considerations proved critical for charge control application in [13].

6 Feedforward Governor

The feedforward governor is a modification of a concept of a reference governor [1, 6] to deal with pointwise-in-time state and control constraints. For simplicity, it is applied here to a disturbance-free (i.e., $w \equiv 0$) system (2.1). In the subsequent treatment we assume that the matrix $g(x_d)$ has full column rank.

The feedforward governor approach is to adjust an offset, $\tilde{u}_d$, to the feedforward term in the controller,

$$u = (u_d + \tilde{u}_d) + u_p + \theta, \quad u_p = -\Pi(L_g Q)^{\mathrm{T}}, \quad \dot{\theta} = -\Gamma(L_g Q)^{\mathrm{T}}$$

to ensure that poitnwise-in-time constraints of the general form

$$(x(t), u(t)) \in C, \quad \forall t$$

are satisfied. The scheme for adjusting $\tilde{u}_d$ can be based on a reference governor-like algorithm of [6]. Since this application of the reference governor is in the context of the stabilization problem and not a tracking problem, and since the governor adjusts the feedforward offset as opposed to the reference command value, the governor is referred to as a *feedforward* governor.

The governor operation is based on sets that are "safe" and "strongly returnable". The "safe" sets are sets of initial conditions for which the ensuing trajectory with constant offset $\tilde{u}_d$ satisfies the pointwise-in-time constraints. With a Lyapunov function of the form

$$V(x, \theta, \tilde{u}_d) = Q(x) + \frac{1}{2}(\theta + \tilde{u}_d)^{\mathrm{T}}(\theta + \tilde{u}_d),$$

the “safe” sets can be selected as subsets of V of the form

$$\Pi(\tilde{u}_d) = \{(x,\theta) : \; V(x,\theta,\tilde{u}_d) - \gamma(\tilde{u}_d) \leq 0\}.$$

If the pointwise-in-time state constraints apply to x only, and $\Pi(\tilde{u}_d^1) \neq \emptyset$ for some $\tilde{u}_d^1$ then $\Pi(\tilde{u}_d) \neq \emptyset$ for any $\tilde{u}_d$. Specifically, if $(x,\theta) \in \Pi(\tilde{u}_d^1)$ then $(x, \theta - \tilde{u}_d + \tilde{u}_d^1) \in \Pi(\tilde{u}_d)$. This suggests that the set $\bigcup_{\tilde{u}_d} \Pi(\tilde{u}_d)$ has naturally a slab-like structure. The situation when there are control constraints can be considerably more complex.

Suppose that the subsets $\Pi(\tilde{u}_d)$ are “safe” for all $\tilde{u}_d \in \mathcal{U}_d$, where $\mathcal{U}_d$ is a compact and convex set with $0 \in int\mathcal{U}_d$. If these subsets have a nonempty interior containing point $(x_d, -\tilde{u}_d)$, and the strong achievability condition holds, it follows that these subsets are “strongly returnable” in the sense of [6]. Specifically, any trajectory originating in $\Pi(\tilde{u}_d)$ with $\tilde{u}_d$ maintained at a constant value is guaranteed to enter in a finite time and stay in the interior of $\Pi(\tilde{u}_d)$.

In each discrete decision step $\tau = 0, 1, 2, \cdots$, at time $t_\tau = \tau \cdot \Delta$, the feedforward governor defines the value of $\tilde{u}_d(t)$ for the interval $t_\tau \leq t < t_{\tau+1}$ by

$$\tilde{u}_d(t) = \bar{u}_d(\tau), \quad t_\tau \leq t < t_{\tau+1}, \; t_\tau = \tau \cdot \Delta, \; \tau = 0, 1, 2, \cdots, \tag{6.19}$$

where

$$\bar{u}_d(\tau+1) = (1 - \kappa(\tau))\bar{u}_d(\tau), \quad 0 \leq \kappa(\tau) \leq 1. \tag{6.20}$$

The parameter $\kappa(\tau)$ is selected so that it is a maximum value from the interval $0 \leq \kappa(\tau) \leq 1$ subject to $(x(t_\tau), \theta(t_\tau)) \in \Pi((1-\kappa(\tau))\bar{u}_d(\tau))$. The initial values $\bar{u}_d(0)$ and $\theta(0)$ are selected so that $(x(0), \theta(0)) \in \Pi(\bar{u}_d(0))$, $\bar{u}_d(0) \in \mathcal{U}_d$.

Thus $\tilde{u}_d(t)$ exhibits a piecewise constant trajectory and its values are defined as a function of system state, integral state of the controller and previous value of the feedforward offset. The feedforward governor attempts to drive $\tilde{u}_d$ to zero as fast as possible (so that nominal control can be administered to the system) while avoinding pointwise-in-time constraint violation. The results in [6] show that $\tilde{u}_d(t) = 0$ for all t sufficiently large and that the region of initial states $x(0)$ that are recoverable by the controller without violating the constraints can be significantly enlarged compared to the situation when $\tilde{u}_d(t) \equiv 0$. The region of recoverable initial conditions can be further enlarged by a proper initialization of the integral state $\theta(0)$.

The procedure for selecting $\kappa(\tau)$ as the maximum possible value in the interval $0 \leq \kappa(\tau) \leq 1$ aims at forcing $\tilde{u}_d(t)$ to zero as soon as possible and it is also essential to guaranteeing that $\kappa(\tau) = 1$ for all τ sufficiently large. It may be possible to define other, also meaningful procedures for selecting $\bar{u}_d(\tau)$.

Suppose, in particular, that the quality of system state trajectories is evaluated using a cost functional of the form

$$J(x(0), \theta(0), \tilde{u}_d)) = \int_0^{+\infty} G(x(t))dt,$$

where $x(t)$ denotes the state trajectory ensuing from the initial condition $\theta(0)$, $x(0)$ and a control trajectory $\tilde{u}_d$. Suppose further that $\tilde{u}_d(\tau) = \bar{u}_d(t) \equiv \bar{u}_d = const$. Under reasonable additional assumptions it can be guaranteed that $x(t) \to x_d$ exponentially, independently of $\bar{u}_d$ and $J(x(0), \theta(0), \tilde{u}_d))$ is finite. While this property holds for the feedforward governor case, it does not hold (in general) in the reference governor: If reference governor output is constant and different from the actual reference input, then quality measures of integral type for the state trajectory deviation from the commanded equilibrium are typically infinite.

We can similarly a candidate control sequence to be applied starting at time τ:

$$\{\bar{u}_d(\tau|\tau), \bar{u}_d(\tau+1|\tau), \cdots, \bar{u}_d(\tau+L|\tau), \cdots\},$$

with $\bar{u}_d(s|\tau) = \bar{u}_d(\tau+L|\tau)$ for $s > \tau + L$. The values $\bar{u}_d(\tau|\tau), \bar{u}_d(\tau+1|\tau), \cdots, \bar{u}_d(\tau+L|\tau)$ can be determined on-line using the model predictive control (MPC) approach. Specifically, at time τ we can *minimize* the value of the cost-to-go subject to original pointwise-in-time constraints and subject to $(x(t_{\tau+L}|\tau), \theta(t_{\tau+L}|\tau)) \in \Pi(\bar{u}_d(\tau+L|\tau))$. Then, we set $\bar{u}_d(\tau) = \bar{u}_d(\tau|\tau)$ and repeat the procedure at time $\tau+1$. Standard MPC properties such as constraint satisfaction and incremental cost convergence to zero, $\int_0^{\Delta} H(x(t))dt \to 0$, can be guaranteed under the usual MPC assumptions. Unlike the feedforward governor (6.19),(6.20), the on-line optimization of the whole offset sequence may be difficult to implement in real-time due to an extensive nature of the required on-line computations.

7 Gasoline direct injection engine applications

The enhancements to the basic Speed-Gradient approach described in this paper have been useful in developing control functionality for gasoline direct injection engines. Two of these applications are now briefly reviewed in more detail.

The approach to transient response shaping of the closed loop system by a proper choice of the weights in the objective function Q has been demonstrated in [9]. Specifically, torque control mode (with torque as the primary tracking objective) or air-to-fuel ratio control mode (with air-to-fuel ratio as the primary tracking objective) can be enforced by proper selection of the weights in Q which is a weighted sum of tracking errors in the torque τ, air-to-fuel ratio, $\lambda = W_{cyl}/W_f$, and spark timing, δ:

$$Q = \frac{1}{2}\gamma_1(\tau - \tau_d)^2 + \frac{1}{2}\gamma_2(W_{cyl} - \lambda_d W_f)^2 + \frac{1}{2}\gamma_3(\delta - \delta_d)^2,$$

where W_{cyl} is the air flow into the engine cylinders, W_f is the fueling rate, and τ_d, λ_d and δ_d are desired set-points for torque, air-to-fuel ratio and spark timing, respectively. Through the augmentation of barrier functions to Q, soft state constraints can be enforced. In [10] this approach was used to enforce state constraints on the admissible range of air-to-fuel ratio and spark timing. Local closed loop stability was demonstrated with the help of a practical numerical procedure that verified the achievability condition directly on a nonlinear system model.

The second application, to charge control in gasoline direct injection engines, has been reported in [13]. In this application, the unmeasured flow through the EGR valve was viewed as an unknown input, and estimated by an input observer. The SG-PI controller was applied to regulate intake manifold pressure and EGR flow estimate to their respective set-points. The design was based on a function Q weighting tracking errors in the intake manifold pressure, p_m, and EGR flow estimate, $\hat{W}_{egr}$:

$$Q = \frac{1}{2}\gamma_1(p_m - p_{m,d})^2 + \frac{1}{2}\gamma_2(\hat{W}_{egr} - W_{egr,d})^2,$$

where $p_{m,d}$ and $W_{egr,d}$ are the desired set-points for the intake manifold pressure and EGR flow rate. The estimate of the EGR flow, $\hat{W}_{egr}$, is generated by an input observer from measurements of intake manifold pressure and throttle flow. The closed loop stability was shown despite the intake to exhaust delay under the assumption of sufficiently strong feedback applied to the electronic throttle to enforce the intake manifold pressure command tracking.

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