# Nonlinear Control in Automotive Engine Applications

Mrdjan Jankovic Ford Research Laboratory P.O. Box 2053, MD 2036 SRL Dearborn, MI 48121 e-mail: mjankov1@ford.com

#### Abstract

Nonlinear control theory has undergone a rapid development in the past decade. Some of these results and ideas have found application in the field of automotive powertrain control where the models that are used are typically multi-input, multi-output, and nonlinear. In this paper, we illustrate a particular application where the disturbance decoupling paradigm is used to design a controller that coordinates the electronic throttle and variable cam timing actuators to achieve a desired transient engine performance. Some aspects of this problem relevant to practical application are also discussed in the paper.

#### 1 Introduction

From the control design point of view, an automotive engine presents a very interesting and very challenging system due to its multiple control inputs and multiple measured outputs, nonlinearities, and presence of external signals and disturbances. Typically, an engine control system is characterized by a hierarchical architecture. At the lowest hierarchy level, various local control systems regulate positions or settings for each actuator such as the electronically controlled throttle, exhaust gas recirculation (EGR) valve, cam phasing actuator, etc. At an intermediate level, the control system is responsible for scheduling and coordinating various engine variables such as spark timing, fuel, throttle opening, EGR valve opening, etc, to achieve best engine performance, fuel economy, and emissions. At the highest level, the control system interprets driver's demand, handles mode transitions, failure mode management, reconfigures the system if necessary, etc.

At the lowest level of control hierarchy, the simple PI and PID controllers are ubiquitous. At the intermediate level the complex interactions and nonlinearities typically make application of more advanced control methods worthy of consideration. Over the years, a number of nonlinear control applications to powertrain systems have been reported in the literature including the feedback linearization [4] and backstepping [2] controllers for powertrains with continuously variable transmission, a feedback linearizing controller for air and fuel control in spark ignition engines [1], a control Lyapunov function based controller proposed in [7] for a diesel engine with variable geometry turbine and EGR, and the speed gradient adaptive control of a direct injection gasoline engine [9].

In this paper we consider the problem of coordinating the electronically controlled throttle (ETC) and variable cam timing (VCT) actuators in controlling the torque of gasoline (spark-ignited) engines [6, 8]. The main challenge is due to the multivariable and nonlinear nature of the plant under control. To illustrate this point, let us consider the steady state air intake in VCT engines where both actuators (the throttle and the VCT) control the amount of air and, in turn, engine torque. The amount of air that the engine pumps out of the intake manifold during each stroke is

determined by the intake manifold pressure  $P_m$ , engine speed N, and position of the VCT actuator  $\zeta_{cam}$ . At a fixed engine speed, which can be considered an external, known variable because it is not under direct control of the engine control system, the cylinder mass air flow  $W_{cyl}$  as a function of manifold pressure is given by straight lines with the slope and offset that depend on cam timing as depicted in Figure 1.



Figure 1: Engine mass air flow versus manifold pressure at different values of cam timings (solid lines) and throttle positions (dash curves).

The dash curves in Figure 1 show the throttle mass air flow rate as a function of the intake manifold pressure (assuming nominal ambient pressure and temperature) and the throttle opening  $\theta$ . In steady state, the flow through the throttle into the intake manifold must be equal to the flow out of the manifold into the engine cylinders. That is, in steady state, the engine operates with the intake manifold pressure and mass air flow determined by the intersection of the engine pumping line that depend on cam timing and the throttle dependent curve. It is clear from Figure 1 that, as the manifold pressure increases, the throttle loses its control authority and cam timing gains authority. Thus, if the cam timing is changed from  $\zeta_{cam_1}$  to  $\zeta_{cam_2}$ , at the throttle opening  $\theta_1$ , there is no change in steady state value of the air-flow (and engine torque). As the throttle opening and manifold pressure increase, the effect (and DC gain) of cam timing on air-flow increases. In contrast, the throttle control authority decreases. At  $\zeta_{cam_2}$ , opening the throttle beyond  $\theta_3$  changes the air-flow very little.

Obviously, even at a low manifold pressure, the cam timing will have an effect on engine air-flow, such as the transient of air-flow due to cam change from  $\zeta_{cam\_1}$  to  $\zeta_{cam\_2}$  at  $\theta = \theta_1$ . In this paper, we shall consider the problem of coordinating the ETC and VCT actuators to deliver a smooth engine torque response to driver's demand and best steady state emissions and fuel economy. The control law derivation and some experimental results can be found in [6, 8]. The original control design was motivated by the disturbance decoupling problem for nonlinear systems considered, for example, in [5]. In this paper we shall emphasize this connection between the control theory and the practical application.

The paper is organized as follows. In Section 2 we review the engine model and discuss the control problem in more details. Section 3 recasts the problem in terms of model reference disturbance decoupling and derives the control law. Section 4 contains the implementation relevant discussion and experimental results.

# 2 Coordination of ETC and VCT actuators

For a dual-equal VCT engine a simple model has been derived in [13] taking a mean-value model of the intake manifold from [12] as the starting point. By differentiating the ideal gas law PV = mRTand neglecting the term due to  $\dot{T}$ , as is standard in SI engine modeling, we obtain the rate of change of manifold pressure

$$\dot{P}_m = K_m (W_\theta - W_{cyl}) \tag{2.1}$$

where  $K_m = \frac{RT_m}{V_m} (T_m, V_m \text{ are manifold temperature and volume, } R$  is the gas constant for air) is assumed constant, and  $W_{\theta}$  is throttle mass flow rate. The mass air flow through the throttle body (c.f. Figure 1) is a function of the upstream atmospheric pressure and temperature, intake manifold pressure, and the throttle angle:

$$W_{\theta} = g_1 \left(\frac{P_m}{P_a}\right) g_2(\theta, P_a, T_a)$$
(2.2)

where

$$g_1(r) = \begin{cases} \gamma^{\frac{1}{2}} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} & \text{if } r \le \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \\ \sqrt{\frac{2\gamma}{\gamma-1}} \left(r^{\frac{2}{\gamma}} - r^{\frac{\gamma+1}{\gamma}}\right) & \text{if } r > \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \end{cases}$$
(2.3)

with  $r = P_m/P_a$ , and  $\gamma = \frac{c_p}{c_v} = 1.4$ . The function  $g_2(\theta, P_a, T_a) = C(\theta) \frac{P_a}{\sqrt{RT_a}}$ , where  $C(\theta)$  is the throttle angle dependent mass air flow characteristic obtained from static engine data. We assume that  $P_a = 1$  bar,  $T_a = 300$  K and suppress the dependence of  $g_1$  and  $g_2$  on  $P_a, T_a$ .

The mass air flow rate into the cylinders can be represented by an affine function of the manifold pressure  $P_m$  as shown in Figure 1. The slope and offset coefficients dependent on cam timing  $\zeta_{cam}$ , and engine speed N:

$$W_{cyl} = \alpha_1(N, \zeta_{cam})P_m + \alpha_2(N, \zeta_{cam})$$
(2.4)

The functions  $\alpha_1$  and  $\alpha_2$  are such that, at given manifold pressure and engine speed, the cam timing effect on  $W_{cyl}$  increases as the cam phase retards. To simplify derivation, we shall assume that the engine speed is slowly varying and suppress the dependence of  $\alpha_1$  and  $\alpha_2$  on N.

Using the engine mapping data, one can determine the optimal (in terms of fuel economy and emissions) schedule of cam timing and throttle opening that produces the desired engine torque at any given engine speed. The question is how to move between the desired set points, as the driver demand and engine speed change, in a manner that minimizes variation from the cylinder air-flow (and torque) response of the conventional (non-VCT) engine.

In comparison with many other regulation problems, this control system is more demanding on the transient performance than on steady state accuracy (though the latter must not be neglected). This suggests that, instead of the set point regulation, we should consider trajectory tracking, with the desired trajectory determined by the conventional engine response. The response that will serve as a reference, corresponds to the cam timing fixed at its nominal (base) value  $\zeta_{cam} = 0$ :

$$\dot{P}_{ref} = K_m(g_1(P_{ref})g_2(\theta_0) - \alpha_1(0)P_{ref} - \alpha_2(0))$$

$$W_{ref} = \alpha_1(0)P_{ref} - \alpha_2(0)$$
(2.5)

where  $P_{ref}$  is the manifold pressure variable for the reference (conventional engine) model, and the "nominal" throttle opening  $\theta_0$  is determined based on driver's demand, engine speed, and possibly other variables, but does not include any compensation for, or coordination with, the VCT actuator. Note that the reference dynamics (2.5) will have to be generated by the control system. An obvious choice is to look at a feedback solution to the problem. There are several obstacles in this direction. First, the actual "performance" variables, the air-charge or engine torque are not measured. The next upstream variable, the manifold pressure can be measured, but the closed loop system bandwidth required to make use of the measured pressure turned out to be very high and would have required high sampling rates. One reason is that, as the pressure approaches the ambient, its dynamics becomes very fast due to the rapid change of the throttle mass air-flow with manifold pressure (dash curves in Figure 1). This approach has not been pursued beyond the simulation stage.

Another important feature of this problem is the difference between the two actuators that control the engine air flow. The ETC is electrically actuated, and is fast, accurate, and repeatable. The VCT mechanism is hydraulically actuated (see [10, 11]) and may be much slower than the ETC. In particular, the speed of response depends on oil pressure which, in turn, may vary significantly with engine speed and oil temperature. Under extreme conditions, the actuator may remain locked in its nominal position, and is not available for engine function control.

With the above facts in mind, we have decided to treat VCT as a known disturbance input, and use the throttle to regulate  $|W_{cyl} - W_{ref}|$  to 0.

### 3 Disturbance decoupling

To design a controller that actuates the throttle to cancel disturbances caused by the VCT, we turn to the problem of disturbance decoupling (c.f. Section 4.6 in [5]). Given a nonlinear system

$$\dot{x} = f(x) + g(x)u + p(x)w$$
  

$$y = h(x)$$
(3.1)

with x being the state, u the control input, and w a disturbance, we consider the problem of finding a control law for u to reject the influence of the disturbance on the output y. Below we assume that the state vector x is available for measurement.

The key role in deciding if the disturbance decoupling problem is solvable is played by the concept of "relative degree" (c.f. [5]). The relative degree, denoted here by r, from an input variable u (or w) to the output y, tells us how many times we need to differentiate the output until the input u (respectively w) appears on the right hand side. Thus, relative degree 1 means that y does not directly depend on u, but that  $\dot{y}$ , does. Differentiating the output of (3.1) we obtain

$$\dot{y} = \frac{\partial h}{\partial x}f(x) + \frac{\partial h}{\partial x}g(x)u + \frac{\partial h}{\partial x}u$$

If  $\frac{\partial h}{\partial x}g(x) \neq 0$  we know that the relative degree from the input u to the output y is 1. Similarly, for the relative degree from w to y. If  $\frac{\partial h}{\partial x}g(x)$  and  $\frac{\partial h}{\partial x}p(x)$  are equal to 0, we can differentiate the output the second time to obtain

$$\ddot{y} = \frac{\partial}{\partial x} \left( \frac{\partial h}{\partial x} f(x) \right) \times \left( f(x) + g(x)u + p(x)w \right)$$

Then the relative degree from u to y would be equal to 2 if  $\frac{\partial}{\partial x} \left( \frac{\partial h}{\partial x} f(x) \right) g(x) \neq 0$  and greater than 2 if  $\frac{\partial}{\partial x} \left( \frac{\partial h}{\partial x} f(x) \right) g(x) = 0$ . The same applies to the relative degree from w to y.

It is clear now that, at least when the disturbance w is not measured, the only hope to have the output y unaffected by the disturbance is to have the relative degree  $r_u$  from u to y be strictly smaller than the relative degree  $r_w$  from the disturbance w to y. If this is the case, the control

input u can be used to cancel the effects of the disturbance by canceling the states affected by the disturbance from the  $r_u$ -th derivative of y. In fact, we can assign any desired dynamics to the output y by an appropriate selection of the input u. In the case when  $r_u = 1$  and  $r_w > 1$ , which is relevant for our VCT derivation, by choosing

$$u = \frac{1}{\frac{\partial h}{\partial x}g(x)} \left(-ay - \frac{\partial h}{\partial x}f(x)\right)$$

we obtain  $\dot{y} = -ay$ .

To apply these ideas to the VCT engine control problem, we first adopt a second order model of the VCT actuator:

$$\begin{aligned}
\xi_1 &= \xi_2 \\
\dot{\xi}_2 &= \psi(\xi_1, \xi_2, w) \\
\zeta_{cam} &= \xi_1
\end{aligned}$$
(3.2)

The presence of the disturbance w in the dynamics of  $\xi_2$  models our uncertainty about effects of external variables, such as engine speed and oil pressure, on the dynamics of the actuator. This model does not have the disturbance affecting directly the first state  $\xi_1$  to allow cleaner derivation of the control law. Our experimental implementation will not be affected by this issue.

Recall now that we have decided to treat the throttle position  $\theta$  as the control input and  $W_{cyl}$  is the output. So the relative degree from the input  $\theta$  to the output  $y = W_{cyl}$  is 1 because

$$\dot{y} = \alpha_1(\zeta_{cam})K_m(g_1(P_m)g_2(\theta) - y) + \left(\frac{\partial\alpha_1}{\partial\zeta_{cam}}P_m + \frac{\partial\alpha_2}{\partial\zeta_{cam}}\right)\xi_2$$
(3.3)

Clearly, we can use  $\theta$  to remove the effects of the disturbance and assign y the desired dynamics. Because our control problem is to track the model reference we compute the derivative of  $y_{ref} = W_{ref}$  to obtain the desired dynamics for (3.3):

$$\dot{y}_{ref} = \alpha_1(0) K_m(g_1(P_{ref})g_2(\theta_0) - y_{ref})$$
(3.4)

The throttle characteristic  $g_2$  is invertible and, therefore, the desired value of the throttle angle input that renders the dynamics of y equal to that of  $y_{ref}$  is given by

$$\theta = g_2^{-1} \left( \frac{\alpha_1(0)}{\alpha_1(\zeta_{cam})} \frac{g_1(P_{ref})g_2(\theta_0)}{g_1(P_m)} - \frac{\left(P_m \frac{\partial \alpha_1}{\partial \zeta_{cam}} + \frac{\partial \alpha_2}{\partial \zeta_{cam}}\right)}{K_m g_1(P_m) \alpha_1(\zeta_{cam})} \xi_2 + \frac{y - \frac{\alpha_1(0)}{\alpha_1(\zeta_{cam})} y_{ref}}{g_1(P_m)} \right)$$
(3.5)

For the experimental implementation we have further simplified the control law (3.5) by setting  $\frac{\alpha_1(0)}{\alpha_1(\zeta_{cam})} = 1$ , in effect allowing the reference dynamics to have the time constant proportional to  $\alpha_1(\zeta_{cam})$  instead of  $\alpha_1(0)$ .

## 4 Implementation and experimental results

Let us return to the hierarchical engine control system description given in the introduction. At the highest "vehicle level controller" the driver acceleration demand is interpreted in terms of desired engine torque. In the intermediate, engine level control system, the desired torque signal is used for scheduling and coordinating engine variables including the ETC position and cam timing. A block diagram of a subsystem relevant for ETC/VCT coordination is shown in Figure 2. Note



Figure 2: Block diagram of the engine control subsystem that coordinates throttle opening and cam timing.

that the cam command  $\zeta_{cam}^c$ , which is sent to the lower level local VCT actuator control loop, and the "conventional" throttle position command  $\theta_0$  are computed without taking into account the actuator interactions. The shaded block implements the control law (3.5), and the reference model (2.5), to produce the throttle command that cancels the VCT "disturbance." The output of this block, denoted by  $\theta^c$ , is the throttle command sent to the local ETC actuator control loop.

To implement the control law (3.5) we need to compute or estimate  $\xi_2$ , the first time derivative of  $\zeta_{cam}$ . One way is to build an estimator that uses a model of the VCT actuator dynamics. While this is certainly possible, the task is made difficult by the variability of the cam response caused by external factors. A simpler way, which does not require a model of the VCT actuator dynamics, is to employ approximate differentiation with the second order Butterworth filter for noise suppression:

$$\xi_2 \approx \frac{\omega_n^2 s}{s^2 + 1.414\omega_n s + \omega_n^2} \zeta_{cam}$$

The noise in the cam timing signal comes from cam torsionals with a typical frequency that is equal to one half the firing frequency. The selection of the filter natural frequency must also take into account the required speed of response of the throttle needed to cancel cam phasing effects on the air-charge and torque. A good range of  $\omega_n$  is determined to be between 20 and 50 rad/s (3-8 Hz).

Another change in the control law implemented experimentally was to replace the measured manifold pressure  $P_m$  in (3.5) with its feedforward estimate computed from the reference cylinder air-flow:

$$\hat{P}_m = \frac{1}{\alpha_1(\zeta_{cam})} (y_{ref} - \alpha_2(\zeta_{cam}))$$

This removes the need to measure the intake manifold pressure and brakes the potential loop – manifold pressure  $\rightarrow$  throttle position (via (3.5))  $\rightarrow$  throttle mass air flow  $\rightarrow$  manifold pressure which may cause problems at high pressure values.

The performance of the coordinating controller has been tested experimentally in a dynamometer test cell. Additional details about experimental setup can be found in [3, 8]. We have first let the throttle reject the torque disturbance caused by step changes in cam timing at fixed operating conditions. Traces of the engine response to cam changes, at constant engine speed and constant  $\theta_0$  are shown by dash curves in Figure 3. The case without ETC/VCT coordination corresponds to using  $\theta^0$  as the throttle command (the constant value given by the green-dash line in the second plot in Figure 3). The second plot from the top shows the coordinating (disturbance rejection) actuation  $\theta - \theta_0$ . Note that the perfect coordination is achieved if the torque does not respond



Figure 3: The response of the VCT engine to cam phase changes with (solid line) and without (dash line) the coordinating controller.

to the cam disturbance because in this case the reference torque output produced by the model without VCT is constant (not shown). Obviously, coordinating control removes most of the cam effects in this case (solid-blue line in the top plot).

A more realistic situation arises if the cam timing and  $\theta_0$  concurrently respond to changes in the desired torque. Such a step change is introduce at t = 0s with the engine speed held constant at 1200 RPM and the response traces are shown in Figure 4. The red-dash curves show the actual engine response with the VCT held at the nominal (0 deg.) value. Hence the red-dot curve at the top plot in Figure 4 represents the ultimate reference model response to replicate. The response of the system with the decoupled ETC/VCT scheduling (green-dash curves) produces a torque response that first hesitates and never reaches the desired steady state value. The latter problem (but not the former) can be solved by taking into account the desired VCT schedule when scheduling  $\theta_0$ . The response of the coordinating ETC/VCT system (blue-solid curves) tracks the torque reference well. The residual oscillations come from the engine speed oscillations, that are due to imperfect dynamometer regulation, coupled with very high almost ambient manifold pressure (bottom plot in Figure 4). Note that a fairly aggressive throttle actuation after t=0.5 seconds produces very little effect on torque response due to the loss of throttle control authority at high manifold pressures.



Figure 4: The response of the engine to the step change in desired torque: red-dot traces –  $\zeta_{cam} \equiv 0$ ; blue-solid – response with the coordinating controller; green-dash – response without coordination.

## 5 Conclusion

Due to its nonlinear, multivariable nature, engine control systems can benefit from application of advanced nonlinear control techniques. In this paper we have shown how the disturbance decoupling results and the notion of relative degree introduced by the geometric theory of nonlinear control can be used to design a controller that coordinates the throttle and the VCT actuator to achieve the desired transient performance. The paper also illustrates the design aspects relevant to experimental implementation of the control law.

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