Identification of Nonlinear Dynamic Systems with Multiple Inputs and Single Output using discrete-time Volterra Type Equations

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Abstract

In this article a theoretical framework is presented about the identification of nonlinear dynamic systems with multiple inputs and single output. Basic considerations about discrete-time Volterra type equations are performed. It is shown how measuring equations can be obtained from nonlinear and arbitrary coupled linear time-invariant transfer functions and nonlinear static functions. The principles of system identification on fundamental basics of the Volterra theory are presented and with a simulation example the results of the presented approach are evaluated.

1 Introduction

Real mechatronic systems are in general highly complex nonlinear dynamic systems. A special class of these systems are Single Input and Single Output (SISO) systems where the term *nonlinear* describes one or more existing nonlinear static functions. If the structure and the linear parameters of the system are well-known then the nonlinear static functions can be identified by Neural Networks within an observer [1]. In the case that structural knowledge and the knowledge about the linear parameters of the system is only partially available this identification method fails and other identification algorithms have to be used. Since most of the existing identification methods are derivatives of the linear difference equation stability can either not or only hardly be proven. Furthermore if the disturbed system's output of the unknown system is fed back to the identification algorithm then the disturbances have negative effects on the identification result [2].

A numerical series to approximate functions with one function variable — the so called Volterra series — became practicable for the identification of nonlinear dynamic SISO systems by applying an adaptation law [3]. A detailed knowledge about the structure of the unknown system is not necessary but reduces the need for computational performance. Together with the learning algorithm stability can be proven. Measuring disturbance influences the adaptation law but not the input of the identification algorithm. Better results can be achieved in comparison to identification techniques based on the difference equation. Identification techniques based on the Volterra theory can be used if the impulse response can be truncated at a finite point of time. Nevertheless much computational performance would be needed to identify such systems without any modifications at the identification algorithm. The use of Orthonormal Base Functions (OBFs) in order to approximate the truncated impulse response reduces the number of unknown parameters enormously. With only little loss of information the truncated impulse response can completely be characterized by an weighted additive overlay of OBFs [4].

A more general class are systems with Multiple Inputs and Single Output (MISO) which also include the class of SISO systems. For those systems the term *nonlinear* becomes double-meaning. On the one hand there may appear nonlinear static functions and on the other hand there also may appear nonlinear couplings between the multiple inputs within a MISO system. In the past years there have only been a few investigations about describing MISO systems on fundamental basics of the Volterra theory. Therefore the identification of nonlinear dynamic systems with multiple inputs and single output using discrete-time Volterra type equations is presented in this article. This article focuses MISO systems built of nonlinear coupled subsystems. Linear coupled subsystems building a MISO system have already been investigated in [5].

2 SISO system description

The discrete-time Volterra equation [3] is displayed in equation (2.1) where $y[k] \in \mathbb{R}$ is the system's output and $u[k-i] \in \mathbb{R}$ is the system's input at different points of time. $\mathrm{g}[i] \in \mathbb{R}$ are the elements of the Volterra kernels and $c \in \mathbb{R}$ is the steady state value. The number of multiplied input variables $u[k-i]$ within each Volterra kernel is a characterism for its order, e.g. $u[k-i_1] \cdot u[k-i_2]$ indicates that the corresponding elements $\mathrm{g}[i_1, i_2]$ build a 2^{nd} order Volterra kernel.

$$\begin{aligned} y[k] = c + \sum_{i_1=0}^{\infty} \mathrm{g}[i_1]\, u[k-i_1] + \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty} \mathrm{g}[i_1, i_2]\, u[k-i_1]\, u[k-i_2] + \ldots + \\ + \sum_{i_1=0}^{\infty} \cdots \sum_{i_q=0}^{\infty} \mathrm{g}[i_1, \ldots, i_q]\, u[k-i_1] \cdots u[k-i_q] \end{aligned} \tag{2.1}$$

In this article systems will be considered whose impulse response can be truncated at a finite point of time. Therefore the upper limit of the sums can be substituted by the integer length of the truncated impulse response $m \in \mathbb{N}^+$. Further if just systems are allowed where the input signal does not directly influence the output [6] then the lower limit of the sums $i = 0$ can be replaced by $i = 1$. Then the discrete-time Volterra equation can be rewritten into a simplified discrete-time Volterra type equation

$$\begin{aligned} y[k] = c + \sum_{i_1=1}^{m} g[i_1]\, u[k-i_1] + \sum_{i_1=1}^{m}\sum_{i_2=1}^{m} g[i_1,i_2]\, u[k-i_1]\, u[k-i_2] + \ldots + \\ + \sum_{i_1=1}^{m} \cdots \sum_{i_q=1}^{m} g[i_1,\ldots,i_q]\, u[k-i_1] \cdots u[k-i_q] \end{aligned} \tag{2.2}$$

Despite these simplifications the number of Volterra kernel elements in equation (2.2) is still too large for the practical use in the field of system identification. Therefore the number of Volterra kernel elements has to be reduced by the introduction of OBFs. With only little loss of information the Volterra kernels can completely be rebuilt by a weighted additive overlay of OBFs. The OBFs used in this article are distorted sinus functions [4] which are defined in equation (2.3). The elements $r[j,i] \in \mathbb{R}$ are orthonormalized by the Cholesky decomposition in order to obtain the orthonormal elements $\widetilde{r}[j,i] \in \mathbb{R}$ [7].

$$r[j,i] = \frac{1}{\sqrt{\frac{m}{2}}} \cdot \sin\left(j \cdot \pi \cdot \left(1 - e^{-\frac{i-0.5}{\zeta}}\right)\right) \quad \text{with} \quad 1 \leq j \leq m_r \quad \text{and} \quad 1 \leq i \leq m \tag{2.3}$$

With the form factor $\zeta \in \mathbb{R}^+$ and the number of distorted sinus functions $m_r \in \mathbb{N}^+$ the OBFs can be adjusted to the dynamics of the system. In figure 1 on the left side orthonormal distorted sinus functions are displayed. By an additive overlay of the weighted OBFs the truncated impulse response can be reconstructed, cf. figure 1 on the right side. In literature other OBFs can be found and used instead of orthonormalized distorted sinus functions, e.g. Laguerre [8] or Kautz functions [9].

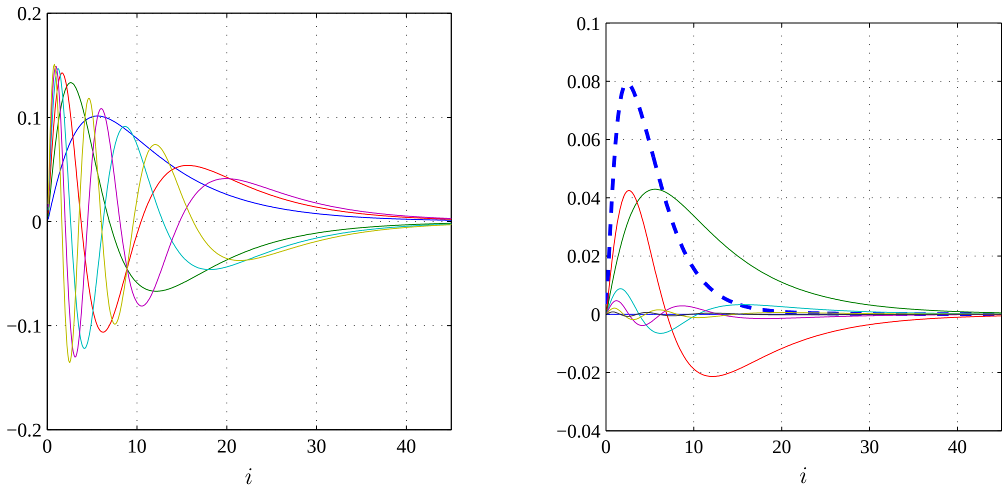

Figure 1: Orthonormal distorted sinus functions (left) and the weighted OBFs reconstructing a truncated impulse response (dashed) by an additive overlay (right).

If the OBFs are applied to equation (2.2) a reduced number of Volterra kernel elements $g_r \in \mathbb{R}$ appears within each Volterra kernel. By inserting the OBFs into equation (2.2) the new Volterra type equation can be written as

$$
\begin{aligned}
y[k] = c &+ \sum_{i_1=1}^{m} \left(\sum_{j_1=1}^{m_r} g_r[j_1]\, \widetilde{r}[j_1, i_1] \right) u[k-i_1] + \\
&+ \sum_{i_1=1}^{m} \sum_{i_2=1}^{m} \left(\sum_{j_1=1}^{m_r} \sum_{j_2=1}^{m_r} g_r[j_1, j_2]\, \widetilde{r}[j_2, i_2]\, \widetilde{r}[j_1, i_1] \right) u[k-i_1]\, u[k-i_2] + \ldots + \\
&+ \sum_{i_1=1}^{m} \sum_{i_2=1}^{m} \cdots \sum_{i_q=1}^{m} \left(\sum_{j_1=1}^{m_r} \sum_{j_2=1}^{m_r} \cdots \sum_{j_q=1}^{m_r} g_r[j_1, \ldots, j_q]\, \widetilde{r}[j_q, i_q] \cdots \widetilde{r}[j_1, i_1] \right) u[k-i_1] \cdots u[k-i_q]
\end{aligned} \tag{2.4}
$$

3 MISO system descriptions

Hammerstein and Wiener models are well-known examples for the identification of SISO systems using discrete-time Volterra type equations. For this reason the following investigations are illustrated by these basic models. The Hammerstein model consists of a nonlinear static function followed by a Linear Time-Invariant (LTI) transfer function as displayed in figure 2 on the left side.

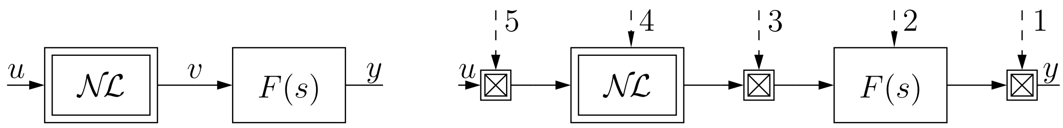

Figure 2: Hammerstein model (left) and five marked points where a second input would may be applied in a nonlinear manner (right).

In this article it is assumed that the nonlinear static function can be described by a polynomial of q^{th} order, i.e. $q \in \mathbb{N}_0^+$. The LTI transfer function will be characterized by the discrete-time convolution of the truncated impulse response and the input. These equations can be written as

$$
v(u) = a_0 u^0 + a_1 u^1 + \ldots + a_q u^q \qquad\qquad y[k] = \sum_{i=1}^{m} h[i]\, v[k-i] \tag{3.1}
$$

By inserting the polynomial into the convolution the mathematical description for a Hammerstein model can be derived

$$
y[k] = a_0 \sum_{i=1}^{m} h[i] + a_1 \sum_{i=1}^{m} h[i]\, u[k-i] + a_2 \sum_{i=1}^{m} h[i]\, u^2[k-i] + \ldots + a_q \sum_{i=1}^{m} h[i]\, u^q[k-i] \tag{3.2}
$$

By a comparison of equation (2.2) with equation (3.2) the following associations can be found. Each Volterra kernel element equals one element in the mathematical description for the Hammerstein model, e.g. $g[i_1] = a_1 \cdot h[i]$ or $g[i_1, \ldots, i_q] = a_q \cdot h[i]$, $\forall i = i_1 = \ldots = i_q$. The Volterra kernels of higher order are just occupied on the main diagonal when describing a Hammerstein model so that they can always be displayed in a two-dimensional space.

In figure 2 on the right side five points indicate where a second input may be applied at the Hammerstein model. In the following paragraphs only the mathematical descriptions for nonlinear coupled systems will be formulated. For system identification these mathematical descriptions have to be transformed into Volterra type equations. In detail, different combinations of polynomial coefficients multiplied with one element of the truncated impulse response equals one Volterra kernel element which are needed for system identification.

If a second input signal may be applied to the Hammerstein model, according to point 5 in figure 2 on the right side, the past-time values of the two input signals at the same point of time have to be multiplied. The nonlinear combined input signals are fed through the nonlinear static function and the LTI transfer function. The mathematical formulation for $r \in \mathbb{N}^+$ input signals is given by

$$\begin{aligned} y[k] = \sum_{i_1=1}^{m_1} h_1[i_1] \Big\{ a_{10} + a_{11} \left(u_1^1[k-i_1] \cdot \ldots \cdot u_r^1[k-i_1] \right) + \ldots + \\ + a_{1q} \left(u_1^q[k-i_1] \cdot \ldots \cdot u_r^q[k-i_1] \right) \Big\} \end{aligned} \tag{3.3}$$

The first indices of each element in the previous and the following formulations is an indication to which input signal the element belongs, e.g. a_{1q} is the q^{th} coefficient of the polynomial which belongs to the first input.

A more general case is the one if an input signal or more are applied at point 4 in figure 2 on the right side. For this case a multi-dimensional polynomial has to be used which is defined for r input signals in the following equation

$$v(u_1, u_2, \ldots, u_r) = \sum_{n_1=0}^{q_1} \sum_{n_2=0}^{q_2} \cdots \sum_{n_r=0}^{q_r} a_{n_1 n_2 \ldots n_r} u_1^{n_1} u_2^{n_2} \cdots u_r^{n_r} \tag{3.4}$$

Different orders in each direction may have to be used for describing a multi-dimensional nonlinear static function. Therefore the upper limits of the sums are defined by $q_1 \ldots q_r \in \mathbb{N}^+$. After inserting equation (3.4) into equation (3.1) the mathematical formulation for this case can be obtained

$$y[k] = \sum_{i_1=1}^{m_1} h_1[i_1] \left\{ \sum_{n_1=0}^{q_1} \sum_{n_2=0}^{q_2} \cdots \sum_{n_r=0}^{q_r} a_{n_1 n_2 \ldots n_r} u_1^{n_1}[k-i_1] \, u_2^{n_2}[k-i_1] \cdots u_r^{n_r}[k-i_1] \right\} \tag{3.5}$$

If an input signal will be applied at point 3 as displayed in figure 2 on the right side it can be shown that this case is already included in equation (3.5). The past time-values of the input signal $u_2[k-i_1]$ will just occur to the power of one. In the case of r input signals applied at the same point all the input signals except $u_1[k-i_1]$ occur to the power of one.

Applying another input signal at point 2 in figure 2 on the right side is not allowed as long as $F(s)$ is a linear system because this would violate the linearity principle and therefore this case will be excluded from the investigations.

As defined in section 2, applying a second input signal at point 1 in figure 2 on the right side this would directly influence the output signal and therefore this coupling is also excluded from the investigations. At first sight it could be meant that equation (3.5) is the most general case for forward directed and nonlinear coupled Hammerstein models but the general case is the one if two or r Hammerstein models will be multiplied at the output. For r input signals this leads to the following mathematical formulation which generally includes the descriptions before

$$\begin{aligned} y[k] = &\left(a_{10} \sum_{i_1=1}^{m_1} h_1[i_1] + a_{11} \sum_{i_1=1}^{m_1} h_1[i_1]\, u_1[k-i_1] + a_{12} \sum_{i_1=1}^{m_1} h_1[i_1]\, u_1^2[k-i_1] + \ldots + \right.\\ &\left. + a_{1q} \sum_{i_1=1}^{m_1} h_1[i_1]\, u_1^{q_1}[k-i_1] \right) \cdot \ldots \cdot \left(a_{r0} \sum_{i_r=1}^{m_r} h_r[i_r] + a_{r1} \sum_{i_r=1}^{m_r} h_r[i_r]\, u_r[k-i_r] + \right.\\ &\left. + a_{r2} \sum_{i_r=1}^{m_r} h_r[i_r]\, u_r^2[k-i_r] + \ldots + a_{rq} \sum_{i_r=1}^{m_r} h_r[i_r]\, u_r^{q_r}[k-i_r] \right) \end{aligned} \tag{3.6}$$

In figure 3 on the left side the cases of nonlinear coupled Hammerstein models for $r = 2$ input signals are displayed which have been investigated yet. The systems which can be described may have one of the dashed couplings displayed. It can also be seen that not all nonlinear couplings of Hammerstein models have been investigated yet. There may also exist systems with backward directed, nonlinear couplings as displayed in figure 3 on the right side.

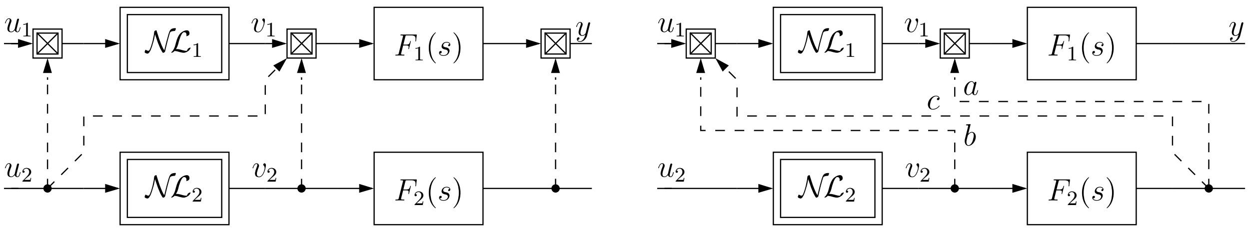

Figure 3: Forward directed nonlinear couplings (left) and backward directed nonlinear couplings (right) between two Hammerstein models.

The mathematical formulations for the cases marked with the letters $a \ldots c$ in figure 3 on the right side can be derived in a similar way as before for nonlinear coupled Hammerstein models in forward direction. Displaying all mathematical formulations for these cases would go beyond the scope of this article because the mathematical descriptions for these cases are implemented in the case marked with the letter c in figure 3 on the right side.

The description for $r-1$ Hammerstein models coupled to another input of a Hammerstein model can be derived by the multiplication of the input signal $u_1[k-i_1]$ with r descriptions for Hammerstein models as displayed in the following equation

$$\begin{aligned} y[k] = a_{10}\sum_{i_1=1}^{m_1} h_1[i_1] + a_{11}\sum_{i_1=1}^{m_1} h_1[i_1] \Bigg\{ u_1[k-i_1] \Bigg(a_{20}\sum_{i_2=1}^{m_2} h_2[i_2] + a_{21}\sum_{i_2=1}^{m_2} h_2[i_2]\, u_2[k-i_2] + \\ + \ldots + a_{2q}\sum_{i_2=1}^{m_2} h_2[i_2]\, u_2^{q_2}[k-i_2] \Bigg) \cdots \Bigg(a_{r0}\sum_{i_r=1}^{m_r} h_r[i_r] + a_{r1}\sum_{i_r=1}^{m_r} h_r[i_r]\, u_r[k-i_r] + \ldots + \\ + a_{rq}\sum_{i_r=1}^{m_r} h_r[i_r]\, u_r^{q_r}[k-i_r] \Bigg) \Bigg\} + \ldots + a_{1q}\sum_{i_1=1}^{m_1} h_1[i_1] \Bigg\{ u_1^{q_1}[k-i_1] \Bigg(a_{20}\sum_{i_2=1}^{m_2} h_2[i_2] + \\ + a_{21}\sum_{i_2=1}^{m_2} h_2[i_2]\, u_2[k-i_2] + \ldots + a_{2q}\sum_{i_2=1}^{m_2} h_2[i_2]\, u_2^{q_2}[k-i_2] \Bigg) \cdots \\ \cdots \Bigg(a_{r0}\sum_{i_r=1}^{m_r} h_r[i_r] + a_{r1}\sum_{i_r=1}^{m_r} h_r[i_r]\, u_r[k-i_r] + \ldots + a_{rq}\sum_{i_r=1}^{m_r} h_r[i_r]\, u_r^{q_r}[k-i_r] \Bigg) \Bigg\} \end{aligned} \tag{3.7}$$

The mathematical formulation for a Wiener model can be derived in a similar way as already explained for the Hammerstein model. The Wiener model consists of a LTI transfer function followed by a nonlinear static function as displayed in figure 4 on the left side. Inserting a discrete-time convolution into a polynomial leads to a more complex SISO system description in comparison to the Hammerstein model.

The previous considerations for MISO systems derived from nonlinear coupled Hammerstein models could also be performed for MISO systems derived from nonlinear coupled Wiener models but it should be noticed that each MISO system derived from nonlinear coupled Wiener models can also be build of nonlinear coupled Hammerstein models. This fact is shown in figure 4 where the system depicted on the right side is already known from the MISO system description of nonlinear coupled Hammerstein models and which equals the scheme of a Wiener model displayed in figure 4 on the left side.

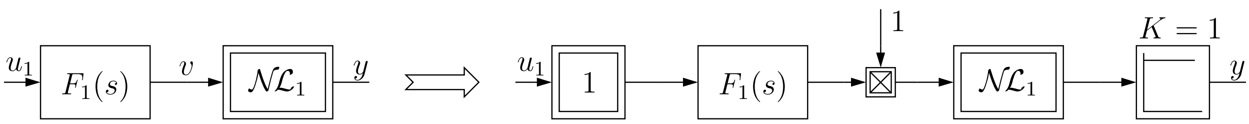

Figure 4: Wiener model (left) as a typical SISO system and the Wiener model consisting of two Hammerstein models in-line (right).

4 Identification algorithm and stability

In general the mathematical description of an unknown system which can be described with the proposed method and whose impulse response can be truncated at a finite point of time, i.e. SISO systems [6] as well as linear [5] and nonlinear coupled MISO systems, can be transformed into the following equation

$$y[k] = \underline{x}^T[k]\,\underline{\theta} + z[k] \tag{4.1}$$

$\underline{\theta} \in \mathbb{R}^{p_r}$ is the parameter vector including the reduced number of Volterra kernel elements of the system and $p_r \in \mathbb{N}^+$. The measuring vector $\underline{x}[k]$ contains m past-time values of the input signal, respectively for MISO systems r input signals and special combinations with different powers from the past-time values of the input signal according to the order of the nonlinear static functions and the structure of the system so that $\underline{x}[k] \in \mathbb{R}^{p_r}$. $z[k] \in \mathbb{R}$ is a disturbance which includes the inherent approximation error, quantization noise and measurement noise. The inherent approximation error would disappear if the upper limit of the truncated impulse response would be substituted by infinity. The noise signals will never disappear because ideal output signal measuring will never be possible. The output signal of the identification algorithm $\widehat{y}[k]$ is defined as

$$\widehat{y}[k] = \underline{x}^T[k]\,\widehat{\underline{\theta}}[k] \tag{4.2}$$

with the unknown parameter vector $\widehat{\underline{\theta}}[k] \in \mathbb{R}^{p_r}$. It can be seen that an equivalent adaptation between an unknown system and the identification algorithm will never be possible as long as the disturbance is present. With the difference between the real and the calculated output an a-posteriori error signal can be defined of the form

$$e_{\text{post}}[k] = y[k] - \widehat{y}[k] = \underline{x}^T[k]\left(\underline{\theta} - \widehat{\underline{\theta}}[k]\right) + z[k] = -\underline{x}^T[k]\,\underline{\phi}[k] + z[k] \tag{4.3}$$

Figure 5 on the left side shows the general identification structure with the unknown system and the identification algorithm. As the adaptation law for online identification the Recursive

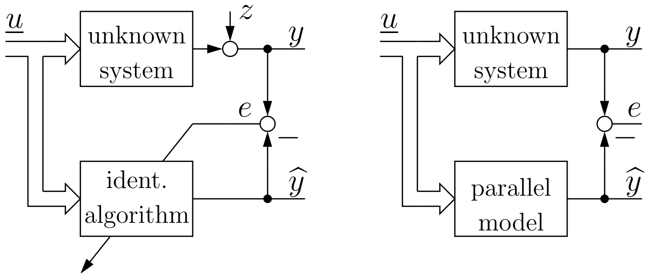

Figure 5: Identification structure for the identification of an unknown systems (left) and validation structure for a qualitative assessment of the identification result (right).

Least SQuare (RLSQ) algorithm is used. On the right side of figure 5 the validation structure is displayed. For the validation process it is assumed that the unknown system is well-known and therefore the disturbance is neglected.

As already pinpointed in the introduction stability can be proven by the use of the Lyapunov stability criterion [10]. For the following introduction into the stability proof the disturbance will be neglected and set to $z[k] = 0, \forall k$. It has to be known that the covariance matrix $\underline{P}[k]$ of the RLSQ algorithm is symmetrical and positive definite at every point of time. Therefore the inverse of the covariance matrix $\underline{P}^{-1}[k]$ does exist and is also positive definite. The a-priori error signal can be calculated with

$$e[k] = \underline{x}^T[k] \left(\underline{\theta} - \widehat{\underline{\theta}}[k-1] \right) = -\underline{x}^T[k]\, \underline{\phi}[k-1] \tag{4.4}$$

This a-priori error signal $e[k]$ is used to adjust the parameters of the parameter vector $\widehat{\underline{\theta}}[k]$. The mathematical formulation for updating the parameter vector $\widehat{\underline{\theta}}[k]$ and for updating the parameter error vector $\underline{\phi}[k]$ can be written as

$$\widehat{\underline{\theta}}[k] = \widehat{\underline{\theta}}[k-1] + \frac{\underline{P}[k-1]\, \underline{x}[k]\, e[k]}{1 + \underline{x}^T[k]\, \underline{P}[k-1]\, \underline{x}[k]} \qquad \underline{\phi}[k] = \underline{\phi}[k-1] + \underline{P}[k]\, \underline{x}[k]\, e[k] \tag{4.5}$$

The Lyapunov function candidate is defined by

$$V\left[\underline{\phi}[k]\right] = \underline{\phi}^T[k]\, \underline{P}^{-1}[k]\, \underline{\phi}[k] \tag{4.6}$$

With the knowledge about the covariance matrix $\underline{P}[k] \in \mathbb{R}^{p_r \times p_r}$ and its inverse $\underline{P}^{-1}[k]$ and with the equations (4.3), (4.4) and (4.5) it can be shown that with the Lyapunov function candidate the following mathematical formulation can be obtained

$$\Delta V\left[\underline{\phi}[k]\right] = V\left[\underline{\phi}[k]\right] - V\left[\underline{\phi}[k-1]\right] = -\frac{e^2[k]}{1 + \underline{x}^T[k]\, \underline{P}[k]\, \underline{x}[k]} \tag{4.7}$$

The nominator and with $\underline{x}^T[k]\, \underline{P}[k]\, \underline{x}[k] \geq 0, \forall k$ also the denominator of the previous equation is always positive. Therefore the whole fraction is negative at every point of time. With $V\left[\underline{\phi}[k]\right] - V[0] \leq 0$ the limes towards infinity can be calculated so that

$$\lim_{k \to \infty} -\frac{e^2[k]}{1 + \underline{x}^T[k]\, \underline{P}[k]\, \underline{x}[k]} = 0 \qquad \Longrightarrow \lim_{k \to \infty} e[k] = 0 \tag{4.8}$$

In other words it can be shown that the limes of the a-priori error tends to zero and therefore the parameter vector of the identification algorithm must tend to the parameter vector of the unknown system if the system is persistently excited and the disturbance is neglected. For this reason the output signal of the identification algorithm is limited at every point of time if the input signal of the identification algorithm is also limited and if the impulse response of the unknown system can be truncated at a finite point of time.

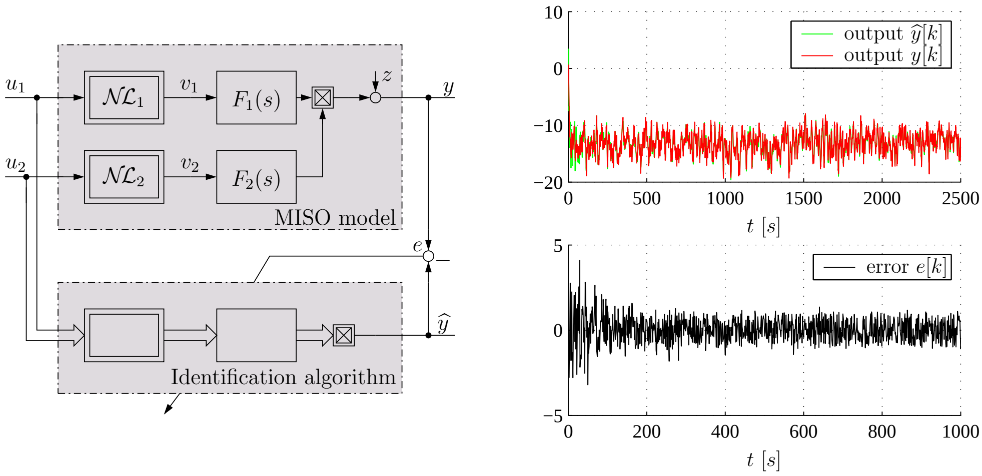

Figure 6: Structure of an unknown MISO system and identification algorithm (left) and the outgoing signals during the identification process (right).

5 Identification example

In this section the identification of two nonlinear coupled Hammerstein models will be illustrated as depicted in figure 6 on the left side. For the identification of this unknown system equation (3.6) has to be transformed into a discrete-time Volterra type equation. As already described for a Hammerstein model in section 3, each combination of a polynomial coefficient multiplied with an element of the truncated impulse response equals an Volterra kernel element. The derived discrete-time Volterra type equation can then be rewritten in form of a measuring equation which can finally be used for the identification of the unknown system. The LTI transfer functions and the nonlinear static functions of the two Hammerstein models are given by

$$F_1(s) = \frac{1}{0.16 + 0.8s + s^2} \qquad v_1(u_1) = 0.1 - 0.3u_1 + 0.5u_1^2$$

$$F_2(s) = \frac{2}{0.16 + 0.4s + s^2} \qquad v_2(u_2) = -0.3 - 0.7u_2 - u_2^2$$

To the system's output of the unknown system a white Gaussian noise signal is added. The identification parameters are chosen as follows

$$r = 2 \qquad h = 0.1s \qquad q_1 = 2 \qquad \frac{m_1}{h} = 350 \qquad \frac{\zeta_1}{h} = 63.6 \qquad m_{r1} = 6$$

$$q_2 = 2 \qquad \frac{m_2}{h} = 400 \qquad \frac{\zeta_2}{h} = 139 \qquad m_{r1} = 8 \qquad \frac{N}{h} = 25000$$

with $h \in \mathbb{R}^+$ as the sampling time and $N \in \mathbb{N}^+$ the value for the identification period of the identification process and the validation process. Without the use of OBFs $p = 641601$ parameters would have to be adapted but with the use of OBFs the reduced number of unknown parameters is $p_r = 221$. In figure 6 on the right side the outgoing signals during the identification process are displayed. At the top of this figure the output signal $y[k]$ and the calculated output signal $\widehat{y}[k]$ are displayed and at the bottom the a-priori error signal $e[k]$ is depicted.

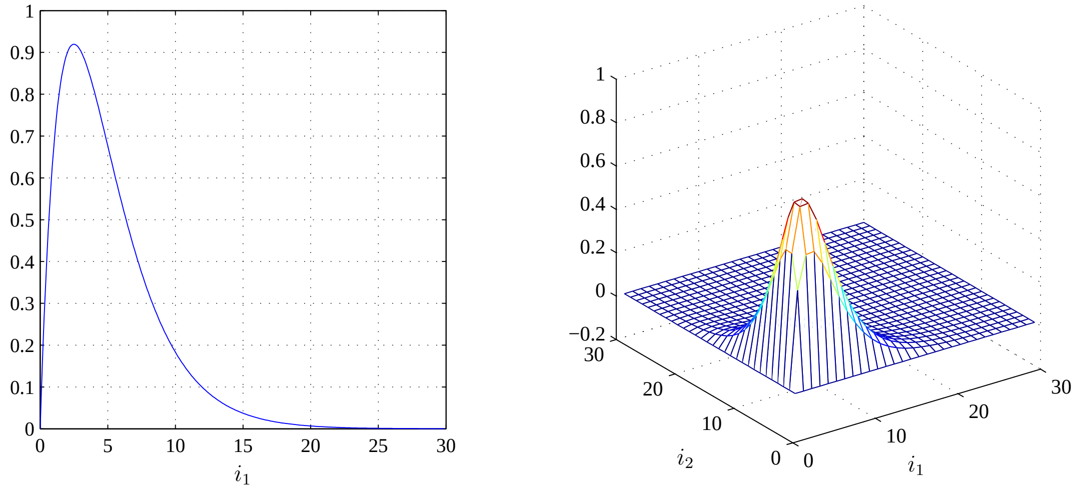

Figure 7: Linear Volterra kernels occur if linear parts are present (left) and two-dimensional symmetrical Volterra kernel occur at special MISO structures (right).

Typical examples of Volterra kernels are displayed in figure 7 and figure 8 on the left side. The linear Volterra kernel always appears if a linear part in a SISO system as well as MISO

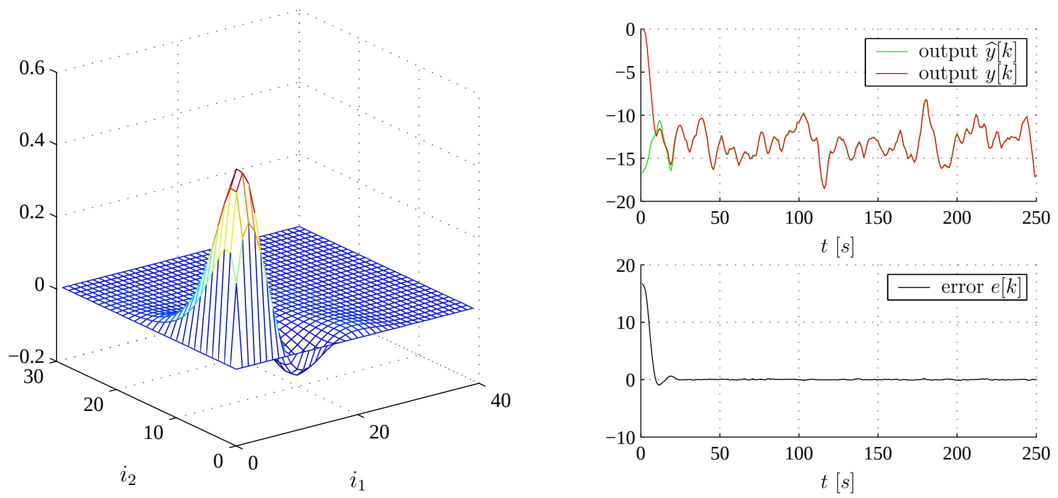

Figure 8: Two-dimensional unsymmetrical Volterra kernel occur at special MISO systems (left) and the outgoing signals during the validation process (right).

system does exist. Two-dimensional symmetrical Volterra kernels occur in MISO systems if the outputs of two LTI transfer functions are multiplied and $F_1(s) = F_2(s)$ or if a LTI transfer function is followed by a nonlinear static function, i.e. Wiener model structures. *The two-dimensional unsymmetrical Volterra kernel* is the result of the multiplication of two output signals from two different LTI transfer functions, i.e. $F_1(s) \neq F_2(s)$.

The result of the undisturbed validation process is depicted in figure 8 on the right side where the output signal of the unknown system $y[k]$, the output signal of the parallel model $\widehat{y}[k]$ and the a-priori error signal $e[k]$ are depicted. At the beginning of the validation process an error is visible because the measuring vector $\underline{x}[k]$ is filled with zeros and therefore a wrong output signal within the parallel model $\widehat{y}[k]$ is calculated. If the measuring vector is completely filled with past-time values of the input signal the output error tends near zero.

6 Conclusion and future aspects

In this article a new approach for the identification of nonlinear dynamic systems with multiple inputs and single output using discrete-time Volterra type equations has been presented. It has been shown that within MISO systems the term *nonlinear* becomes double-meaning, i.e. nonlinear static functions as well as nonlinear couplings within a MISO system may exist. Basic considerations about the Volterra theory have been performed and the mathematical descriptions for different nonlinear coupled MISO systems have been presented. The identification structure as well as basic considerations about the stability proof have been shown. Finally an identification example of nonlinear coupled Hammerstein models and its arising results have been displayed.

Further work has to be done in code optimization due to the fact that still a high number of parameters is needed and therefore a lot of computational performance. The unification of the proposed system identification method with the existing observer theory is planned for the observation of globally integrating MISO systems [11]. Furthermore the developed basics will be used for system identifications in the field of mechatronic applications.

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