Discrete-time modeling and analysis of pulse-width-modulated switched power converters

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Abstract

The subject of this paper is a general theory for a switched power converter, where the switch is controlled according to a pulse-width-modulation strategy. The pulse-width-modulated switched power converter is transformed into an equivalent discrete-time system. With this system the existence, uniqueness and stability of stationary solutions is investigated. Also stability of deviations from a stationary solution is investigated. Furthermore, feedback control situations are considered. It is shown that two kinds of modulators, namely the running and fixed modulator, may invoke a very different behaviour of the feedback control system. The phenomenon of subharmonic oscillations is shown to be a special case of the normal unstable behaviour.

1 Introduction

In this paper a switched power converter (SPC) with one switch is considered. The switch is controlled with a pulse-width modulator (PWM). The state of the switch is completely determined by an input $s(t)$. In fact, we have the situation as depicted in Figure 1. The output of the SPC is

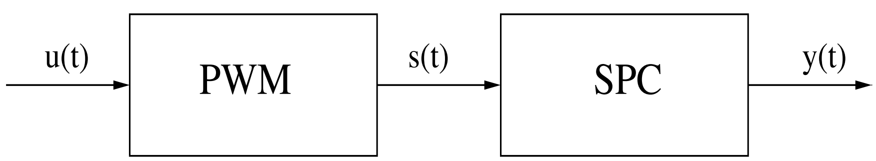

Figure 1: Pulse-width-modulated switched power converter.

$y(t)$ and the input of the modulator is $u(t)$.

In this paper we consider, as example, the Buck power converter. Much has been done already in this field. General theory can be found in [1-2]. Early contributions are [3-4], an elementary contribution is [5] and recent contributions involving chaos are [6-7]. In this paper we will generalize all these references.

The organization of the paper is as follows. In section 2 we consider the modeling of a pulse-width-modulated switched power converter. In section 3 we transform the pulse-width-modulated power converter to an equivalent discrete-time system and investigate existence, uniqueness and stability of stationary solutions. In section 4 the equivalent discrete-time system is linearized. Here stability of deviations from a stationary solution is investigated. In section 5 feedback control situations are considered. We will show that the phenomenon of subharmonic oscillations is just a special case of a normal unstable behaviour. In this section we will also give examples to illustrate the theory. The paper concludes with the conclusions in section 6 and the references.

2 Switched power converters and pulse-width modulators

We consider the Buck converter which is represented in Figure 2. With this converter it is possible to transform *losslessly* the source DC voltage $e_s(t)$ into another DC voltage $e_C(t)$ over the load R. Define the pulse period T. Introduce $L' = L/T,\ C' = C/T$, and for some variable $v(t),\ \mathring{v}(\tau) = T\dot{v}(t)$

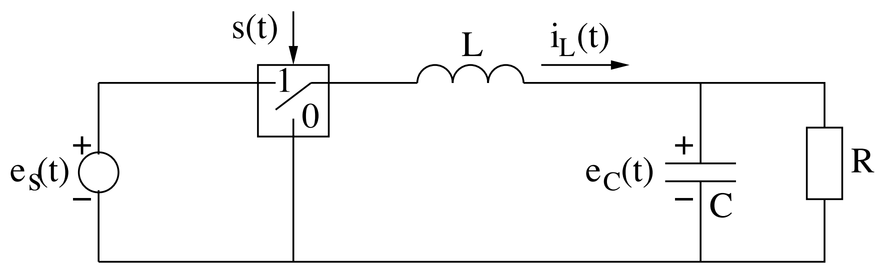

Figure 2: Buck converter.

where $\dot{v}(t)$ denotes the usual derivative of $v(t)$ with respect to t and where $\tau = t/T$ is the normalized time with respect to T. Assume in the Buck converter that the coil current is always positive and that e_s is constant.

It will appear that power converters with one switch and one output can be described by

$$\begin{aligned} \mathring{x}(\tau) &= (A_0 + A_1 s(\tau))x(\tau) + b_0 + b_1 s(\tau), \\ y(\tau) &= cx(\tau), \end{aligned} \tag{1}$$

where $s(\tau)$ is a switch function which values 0 or 1. The matrices $A_0, A_1 \in \mathbb{R}^{n\times n}$, the vectors $b_0, b_1 \in \mathbb{R}^n$, and the vector $c \in \mathbb{R}^{1\times n}$ are constant. Furthermore, we have the state $x(\tau) \in \mathbb{R}^n$, the input $s(\tau) \in \mathbb{R}$ and the output $y(\tau) \in \mathbb{R}$. System (1) is also called a switched linear system (SLS).

The parameters for the Buck converter are as follows.

$$\begin{aligned} A_0 &= \begin{bmatrix} -\frac{1}{RC'} & \frac{1}{C'} \\ -\frac{1}{L'} & 0 \end{bmatrix}, \quad b_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \\ A_1 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 0 \\ \frac{e_s}{L'} \end{bmatrix}, \end{aligned} \tag{2}$$

with $x(\tau) = (e_C(\tau)\ i_L(\tau))^T$.

The switch is controlled with a PWM with input $u(\tau) \in \mathbb{R}$ and output $s(\tau) \in \mathbb{R}$. Essentially there are two kinds of pulse-width modulators, namely

-**Running Modulator**. Here $u(\tau)$ is compared with the sawtooth

$$z(\tau) = z_m \cdot (\tau - k), \quad k \le \tau < k+1, \quad k = 0, 1, \cdots, \tag{3}$$

where z_m is the maximum value of the sawtooth. The pulse duration d_k is determined implicitly while running through the period by

$$z_m \cdot d_k = u(k + d_k). \tag{4}$$

-Fixed Modulator. Here the pulse duration d_k is determined explicitly at the fixed normalized time points $k,\ k = 0, 1, \cdots$, by scaling $u(k)$ with a scaling factor f_s, so

$$d_k = \tfrac{1}{f_s} u(k). \tag{5}$$

3 Equivalent discrete-time system

Define

$$\begin{aligned} &x_k^\delta = x(k+\delta), \quad x_k = x_k^0, \\ &y_k^\delta = y(k+\delta), \quad y_k = y_k^0, \\ &u_k^\delta = u(k+\delta), \quad u_k = u_k^0, \\ &\Phi_s^\delta = e^{(A_0 + A_1 s)\delta}, \\ &\gamma_s^\delta = \textstyle\int_0^\delta e^{(A_0+A_1 s)\mu}(b_0 + b_1 s) d\mu, \end{aligned} \tag{6}$$

where $0 \leq \delta \leq 1$ and $s = 0, 1$. Remark that $\Phi_s^0 = I$ and $\gamma_s^0 = 0$. The input $s(\tau)$ is pulse-width modulated, namely

$$s(\tau) = \begin{cases} 1, & k \leq \tau < k + d_k, \\ 0, & k + d_k \leq \tau < (k+1). \end{cases} \tag{7}$$

The equivalent discrete-time description is based on the transition from x_k to x_{k+1}. Added to this is the determination of $x_k^\alpha,\ 0 \leq \alpha \leq 1$, given a certain x_k. Together we have a complete discrete-time description of the original continuous-time system.

The state x_k is transformed in two transitions to x_{k+1}, namely $x_k \stackrel{1}{\longrightarrow} x_k^{d_k} \stackrel{2}{\longrightarrow} x_{k+1}$:

$$\begin{aligned} x_k^{d_k} &= \Phi_1^{d_k} x_k + \gamma_1^{d_k}, \\ x_{k+1} &= \Phi_0^{1-d_k} x_k^{d_k} + \gamma_0^{1-d_k}. \end{aligned} \tag{8}$$

These expressions are just the solutions of system (1) with the appropiate initial conditions and value of $s(t)$ using (7). Taking transitions (8) together, we have

$$x_{k+1} = \Phi_0^{1-d_k} \Phi_1^{d_k} x_k + \Phi_0^{1-d_k} \gamma_1^{d_k} + \gamma_0^{1-d_k}. \tag{9}$$

Now we have a dynamical description of x_k. Suppose we want to know the value $x_k^\alpha,\ 0 \leq \alpha \leq 1$, then we may distinguish two cases, namely $0 \leq \alpha \leq d_k$ and $d_k \leq \alpha \leq 1$. Firstly we assume x_k given and $0 \leq \alpha \leq d_k$. The state x_k is transferred in one transition to x_k^α, namely $x_k \stackrel{1}{\longrightarrow} x_k^\alpha$:

$$x_k^\alpha = \Phi_1^\alpha x_k + \gamma_1^\alpha. \tag{10}$$

Secondly we assume x_k is given and $d_k \le \alpha \le 1$. The state x_k is transferred in two transitions to x_k^α, namely $x_k \xrightarrow{1} x_k^{d_k} \xrightarrow{2} x_k^\alpha$:

$$
\begin{aligned}
x_k^{d_k} &= \Phi_1^{d_k} x_k + \gamma_1^{d_k}, \\
x_k^\alpha &= \Phi_0^{\alpha-d_k} x_k^{d_k} + \gamma_0^{\alpha-d_k}.
\end{aligned}
\tag{11}
$$

Taking the transitions (11) together we have

$$
x_k^\alpha = \Phi_0^{\alpha-d_k}\Phi_1^{d_k} x_k + \Phi_0^{\alpha-d_k}\gamma_1^{d_k} + \gamma_0^{\alpha-d_k}. \tag{12}
$$

Summarizing (9), (10) and (12) we have

$$
\begin{aligned}
x_{k+1} &= F(d_k)x_k + g(d_k), \\
x_k^\alpha &= P^\alpha(d_k)x_k + q^\alpha(d_k), \\
y_k^\alpha &= c x_k^\alpha,
\end{aligned}
\tag{13}
$$

where $k = 0, 1, \cdots$, and

$$
\begin{aligned}
F(d_k) &= \Phi_0^{1-d_k}\Phi_1^{d_k}, \\
g(d_k) &= \Phi_0^{1-d_k}\gamma_1^{d_k} + \gamma_0^{1-d_k},
\end{aligned}
\tag{14}
$$

and for $0 \le \alpha \le d_k$

$$
\begin{aligned}
P^\alpha(d_k) &= \Phi_1^\alpha, \\
q^\alpha(d_k) &= \gamma_1^\alpha,
\end{aligned}
\tag{15}
$$

and for $d_k \le \alpha \le 1$

$$
\begin{aligned}
P^\alpha(d_k) &= \Phi_0^{\alpha-d_k}\Phi_1^{d_k}, \\
q^\alpha(d_k) &= \Phi_0^{\alpha-d_k}\gamma_1^{d_k} + \gamma_0^{\alpha-d_k}.
\end{aligned}
\tag{16}
$$

Now the original pulse-width-modulated SPC (1) has been transformed into an equivalent discrete-time SPC (EDSPC) given by (13). In fact, we have the situation as depicted in Figure 3. Compare Figure 3 with Figure 1. Note that $s(t)$ is replaced by d_k. Indeed, we may identify

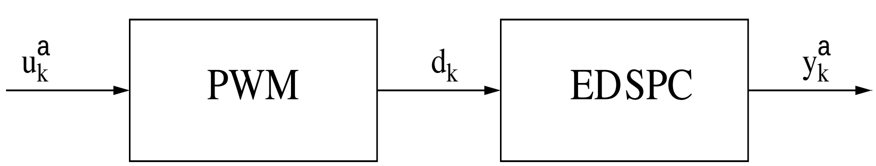

Figure 3: Equivalent discrete-time system.

the functional $\{s(t),\ 0 \le t \le (k+1)T\}$ with the sequence $\{d_0, \cdots, d_k\}$ in the sense that they contain the same information. Remark that the deduction of the EDSPC holds for any SLS, not only power converters.

With the EDSPC (13) we have an equivalent discrete-time description of the SPC (1), in the sense that for a certain k and α, $x(t) = x_k^\alpha = x(kT + \alpha T)$. Choosing $k = 0, 1, \cdots$, and $0 \le \alpha \le 1$, we are able to calculate $x(t)$ for any $t \geq 0$, using the EDSPC (13) and the sequence $d_0, d_1, \cdots, d_k$. That also means that any property related to input/state/output of the continuous-time SPC (1) is equivalent to the corresponding property of the EDSPC (13). The SPC (1) is a continuous-time system with a discontinuous input, so difficult to handle. The EDSPC (13) is a completely discrete-time system, hence much easier to handle. Summarizing we may investigate global properties of (1) via (13).

We introduce the following definition.

Definition 1 *For $d_k = d$, a solution of the EDSPC (13) is called stationary if x_k^α is constant for each α, i.e. $x(t)$ is periodic with period T. This solution is denoted by (d, x^α).*

Assume a stationary solution (d, x^α), then from (13) we have

$$
\begin{aligned}
x &= F(d)x + g(d), \\
x^\alpha &= P^\alpha(d)x + q^\alpha(d), \qquad (17) \\
y^\alpha &= cx^\alpha,
\end{aligned}
$$

which yields the stationary solution x^α given d, i.e.

$$
\begin{aligned}
x &= (I - F(d))^{-1} g(d), \\
x^\alpha &= P^\alpha(d)x + q^\alpha(d), \qquad (18) \\
y^\alpha &= cx^\alpha.
\end{aligned}
$$

It is convenient to introduce the following definition.

Definition 2 *A real square matrix X is called continuous-time stable (CT-stable) if the eigenvalues of X are in the complex open left half plane, and discrete-time stable (DT-stable) if the eigenvalues of X are in the complex open unit disc.*

Then we have the following theorem.

Theorem 1 *Assume that d is given, $0 \leq d \leq 1$, and $F(d)$ is DT-stable. Then there exists a unique stationary solution (d, x^α) of the EDSPC (13), given by (18) for each $\alpha \in [0, 1]$, and this stationary solution is asymptotically stable.*

Proof If $F(d)$ is DT-stable then the inverse in (18) exists and is unique. So there exists a unique stationary solution (d, x^α). Furthermore, the EDSPC (13) is affine in x_k. Hence, if $d_k = d$ and $F(d)$ is DT-stable, then the stationary solution (d, x^α) of the EDSPC (13) is asymptotically stable.

For the Buck converter theorem 1 holds.

4 Linearized system

Consider the EDSPC given by (13), where $F(d_k)$, $g(d_k)$, $P^\alpha(d_k)$ and $q^\alpha(d_k)$ are given by (14-16). Also we will need the formula

$$
(A_0 + sA_1)\gamma_s^\delta = (\Phi_s^\delta - I)(b_0 + sb_1), \quad s = 0, 1, \qquad (19)
$$

which is well known from system theory.

Now assume the stationary solution $(d, x^\alpha),\ 0 \le \alpha \le 1$. As we will see, as far as linearization is concerned, we need only x_k^α or x^α for $0 \le \alpha \le d$. Define the deviations

$$
\begin{aligned}
\Delta x_k^\alpha &= x_k^\alpha - x^\alpha, \\
\Delta d_k &= d_k - d, \\
\Delta y_k^\alpha &= y_k^\alpha - y^\alpha.
\end{aligned}
\tag{20}
$$

Linearizing (13) around (d, x^α) yields

$$
\begin{aligned}
\Delta x_{k+1} &= F(d)\Delta x_k + \left(\frac{\partial F(d_k)}{\partial d_k}\Big|_d x + \frac{\partial g(d_k)}{\partial d_k}\Big|_d \right) \Delta d_k, \\
\Delta x_k^\alpha &= P^\alpha(d)\Delta x_k + \left(\frac{\partial P^\alpha(d_k)}{\partial d_k}\Big|_d x + \frac{\partial q^\alpha(d_k)}{\partial d_k}\Big|_d \right) \Delta d_k, \\
\Delta y_k^\alpha &= c\Delta x_k^\alpha.
\end{aligned}
\tag{21}
$$

From (14), and using (6), we may calculate

$$
\begin{aligned}
\frac{\partial F(d_k)}{\partial d_k}\Big|_d &= \Phi_0^{1-d} A_1 \Phi_1^d, \\
\frac{\partial g(d_k)}{\partial d_k}\Big|_d &= \Phi_0^{1-d}(A_1 \gamma_1^d + b_1).
\end{aligned}
\tag{22}
$$

Furthermore $P^\alpha(d_k)$ and $q^\alpha(d_k),\ 0 \le \alpha \le d$, are independent of d_k, thus the partial derivatives with respect to d_k are zero.

Summarizing we have for $0 \le \alpha \le d$,

$$
\begin{aligned}
\Delta x_{k+1} &= F(d)\Delta x_k + h(d)\Delta d_k, \\
\Delta x_k^\alpha &= P^\alpha(d)\Delta x_k, \\
\Delta y_k^\alpha &= c\Delta x_k^\alpha,
\end{aligned}
\tag{23}
$$

where, using (19),

$$
\begin{aligned}
F(d) &= \Phi_0^{1-d}\Phi_1^d, \\
h(d) &= \Phi_0^{1-d}(A_1 x^d + b_1), \\
P^\alpha(d) &= \Phi_1^\alpha.
\end{aligned}
\tag{24}
$$

System (23) is linear, time-independent and discrete-time, and constitutes the linearized EDSPC (LEDSPC).

The stationary solution $(d, x^\alpha), 0 \le \alpha \le 1$, of the EDSPC corresponds to a constant realitive pulse width d and a periodic $x(\tau), 0 \le \tau \le 1$, of the time-normalized SPC as in (1). Assume that the relative pulse width results from a periodic $u(\tau)$, corresponding to a constant u^α in the EDSPC. Define the deviation

$$
\Delta u_k^\alpha = u_k^\alpha - u^\alpha. \tag{25}
$$

Again we consider separately the running and the fixed modulator.
-**Running Modulator**. Repeating (4) we have $z_m d_k = u_k^{d_k}$. Now observe that

$$z_m \Delta d_k = z_m(d_k - d) = u_k^{d_k} - u^d. \tag{26}$$

The latter can be written as $u_k^{d_k} - u^{d_k} + u^{d_k} - u^d$. The terms $u_k^{d_k} - u^{d_k}$ and $u^{d_k} - u^d$ can be approximated linearly by $u_k^d - u^d$ and $\mathring{u}^d(d_k - d)$, respectively. See also Figure 4. This yields

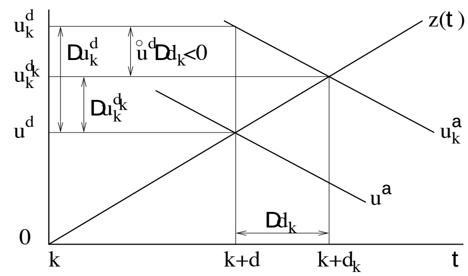

Figure 4: Linearization of the running modulator

approximately

$$z_m \Delta d_k = u_k^d - \mathring{u}^d \Delta d_k. \tag{27}$$

Hence linearizing around (u^d, d) gives

$$\Delta d_k = \tfrac{1}{z_m - \mathring{u}^d} \Delta u_k^d = G_m \Delta u_k^d. \tag{28}$$

-**Fixed Modulator**. Repeating (5) we have $f_s d_k = u_k^0$ or $d_k = u_k^0 / f_s$. Hence linearizing around (u^0, d) gives

$$d_k = \tfrac{1}{f_s} \Delta u_k = G_m \Delta u_k. \tag{29}$$

Assume that $G_m \geq 0$. Equations (28) and(29) give the linearized PWM (LPWM). The LEDSPC (23) represents the dynamic behaviour for small deviations with respect to the stationary solution (d, x^α), $\alpha \in [0,1]$, of the EDSPC (13), and equivalently the SPC (1). Summarizing, we may investigate local properties of (1) via (23). We may state the following theorem.

Theorem 2 *The stationary solution (d, x^α) of the EDSPC (13), and equivalently the SPC (1), is asymptotically stable if and only if the LEDSPC (23) around (d, x^α) is asymptotically stable.*

Proof The system matrices of the EDSPC (13) and the LEDSPC (23) are both $F(d)$. Then with theorem 1 the result follows.

5 Feedback control

Our starting point is the LEDSPC (23). In Figure 5 the feedback control scheme is represented. The controller should be such that the deviations are as small as possible. The controller may be static or dynamic, and can be chosen by the designer. The pulse-width modulator is, for deviations, static with gain G_m, which value depends on the modulation method (28,29). Much can be said about the controller, however in this paper we are only interested in intrinsic properties of the pulse-width-modulated SPC's, without or with feedback. Those properties can be studied properly by taken the controller to be static with gain $-G_c$, $G_c \geq 0$. Define the gain G by

$$G = G_m G_c. \tag{30}$$

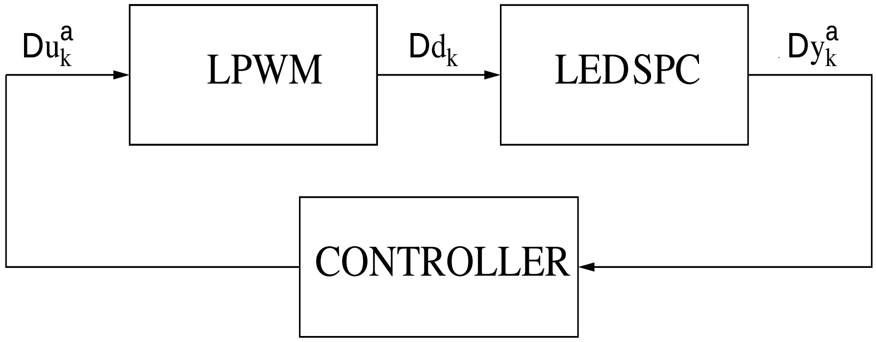

Figure 5: Feedback control scheme.

From (23) we have

$$\Delta x_{k+1} = F(d)\Delta x_k + h(d)\Delta d_k,$$
$$\Delta y_k^\alpha = c\Delta x_k^\alpha = c\Phi_1^\alpha \Delta x_k = c^\alpha \Delta x_k, \quad c^\alpha = c\Phi_1^\alpha, \tag{31}$$

which is denoted by the triple $(F(d), h(d), c^\alpha)$. From Figure 5 we see

$$\Delta d_k = -G_m G_c \Delta y_k^\alpha = -Gc^\alpha \Delta x_k. \tag{32}$$

Inserting Δd_k in (31) gives

$$\Delta x_{k+1} = (F(d) - h(d)Gc^\alpha)\Delta x_k. \tag{33}$$

Here we may distinguish two cases with respect to the modulator, namely
-**Running Modulator**. The controller information is Δy_k^d, so $\alpha = d$, and $c^\alpha = c^d = c\Phi_1^d$.
-**Fixed Modulator**. The controller information is Δy_k, so $\alpha = 0$, and $c^\alpha = c^0 = c\Phi_1^0 = c$.

Now we may draw the root locus of (33), i.e. the eigenvalues of the system matrix in the complex plane, for $G : 0 \longrightarrow \infty$. Assume that $F(d)$ is DT-stable, thus the root locus starts within the unit circle. Suppose that the $\times$'s in Figure 6 indicate the place where the root locus leaves the unit circle for the associated G^*. For $G = G^*(1 + \epsilon)$, where $0 < \epsilon << 1$, the linearized system

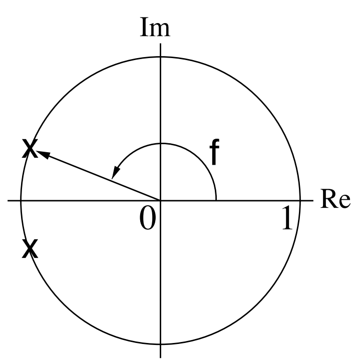

Figure 6: Crossing root locus with unit circle.

will become unstable and, according to theorem 2, also the stationary solution (d, x^α) of the real converter. The instability of the linearized system is unbounded, whereas the instability within the real converter remains bounded. This phenomenon occurs more often in practice and is then due to the nonlinearities present in the system. Here it turns out that the instability contains an extra oscillation and that the boundedness of the instability is such that the d_k remains in the interval $(0,1)$, i.e. does not become saturated. The period T_i of the additional oscillation can be determined from the angle ϕ at which the root locus crosses the unit circle using

$$e^{j2\pi\frac{T}{T_i}} = e^{j\phi}. \tag{34}$$

Hence

$$\eta = \frac{T}{T_i} = \frac{\phi}{2\pi}. \tag{35}$$

Thus the *continuous-time* periodic behaviour is the *superposition of two periodic signals* with respectively the periods T and T_i. From (34) we have

$$\phi : 0 \longrightarrow \pi \quad \Rightarrow \quad \eta : 0 \longrightarrow \tfrac{1}{2}. \tag{36}$$

For $\phi = \pi$ we have exactly the first subharmonic oscillation with period $2T$. This shows that, in spite of the literature in the past [3-4] and recently [6-7], the phenomenon of subharmonic oscillations is **just a special case** of a normal unstable behaviour.

Now we assume $z_m = 1,\ f_s = 1,\ e_s = 1,\ d = 0.5,\ R = 2$, and $y = e_C$. In a stationary situation the variables e_C and i_L have so-called relative ripples

$$\begin{aligned} \rho_{e_C} &= \frac{\hat{e}_C - \check{e}_C}{\bar{e}_C}, \\ \rho_{i_L} &= \frac{\hat{i}_L - \check{i}_L}{\bar{i}_L}, \end{aligned} \tag{37}$$

where $\hat{x}$, $\check{x}$ and $\bar{x}$ denotes respectively the maximum, minimum and average of x. From these relative ripples, and R, the components L' and C' may be determined, approximately, with

$$\begin{aligned} L' &= \frac{(1-d)R}{\rho_{i_L}}, \\ C' &= \frac{1-d}{8L'\rho_{e_C}}. \end{aligned} \tag{38}$$

which can easily be derived from [5]. Assume the relative ripples $\rho_{e_C} = 0.01$ and $\rho_{i_L} = 0.1$. That leads to the L' and C' given in Table 1.

Table 1: L' and C' for the Buck converter.

Converter	L'	C'
Buck	10	0.625

Now we may determine for the Buck converter the root locus as function of G. For example the root loci for the Buck converter with running and fixed modulation are depicted in Figure 7-8.
The G^* where the root locus leaves the unit circle can be determined computationally, also G_m from (28) or (29), and finally the associated G_c^* from (30). In Table 2 these values are given. Here RM denotes running modulator and FM fixed modulator. The following remarks can be made:
1. For the running modulator $G^* \neq G_c^*$ which is immediate from (28), since $G_m \neq 1$.
2. For the fixed modulator $G^* = G_c^*$ which is immediate from (29), since $f_s = 1$.

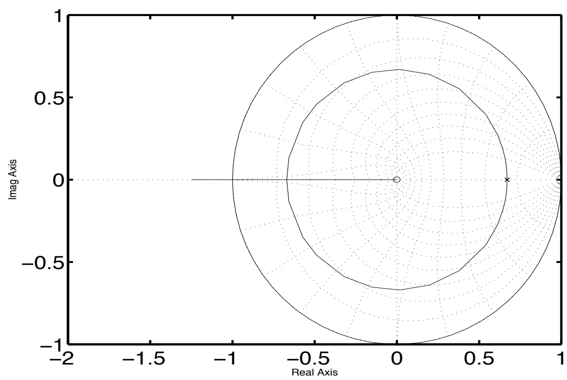

Figure 7: Root locus for the Buck converter and running modulator

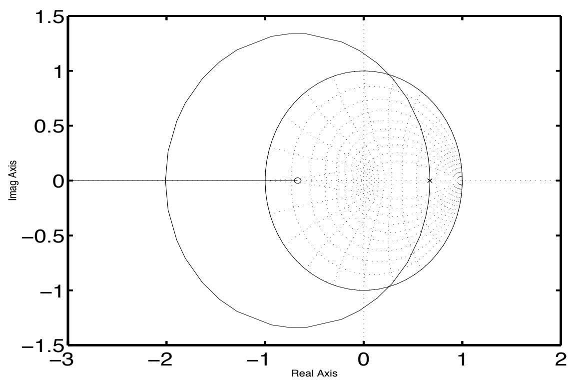

Figure 8: Root locus for the Buck converter and fixed modulator

Table 2: G^* and G_c^* for the Buck converter.

	RM	RM	FM
Converter	G^*	G_c^*	$G^* = G_c^*$
Buck	26	53.6	12.6

Table 3: η for the Buck converter.

Converter	RM	FM
Buck	0.5	0.2

From the root loci of the Buck converter we may also determine, via (35), the value η. The values are given in Table 3 for running and fixed modulation.

Now we may simulate the real Buck converter using the equation

$$u(\tau) = u^\alpha - G_c(y(\tau) - y^\alpha), \tag{39}$$

where $\alpha = 0$ for fixed modulation and $\alpha = d$ for running modulation. The stationary solution of e_C of the simulated real Buck converter is depicted in Figure 9, where G_c is choosen to be

$G_c^*(1-\epsilon),\ 0 < \epsilon << 1$. The relative ripple is 0.01 as assumed. Choosing $G_c = G_c^*(1+\epsilon),\ 0 < \epsilon << 1$,

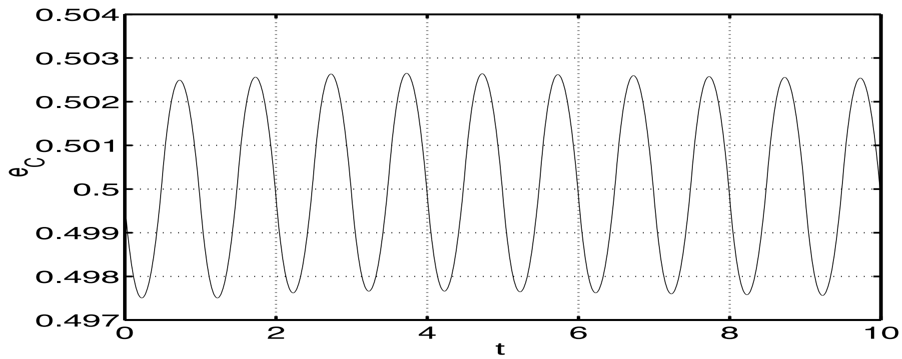

Figure 9: Stationary solution of the real Buck converter with running or fixed modulation.

we get unstable periodic solutions which are depicted in Figure 10-11. The effects predicted by

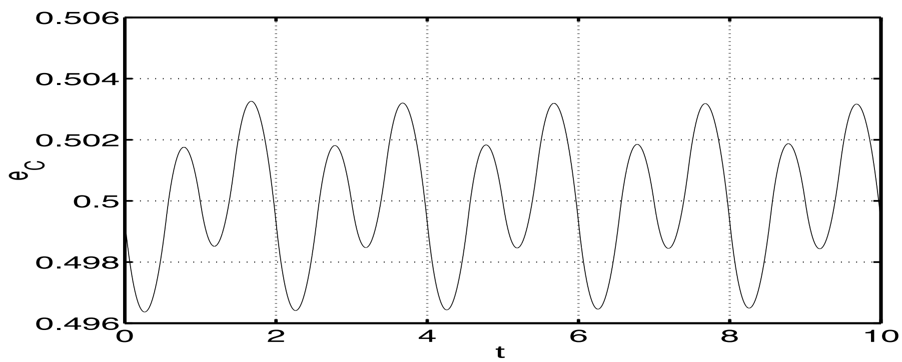

Figure 10: Unstable periodic solution of the real Buck converter with running modulation.

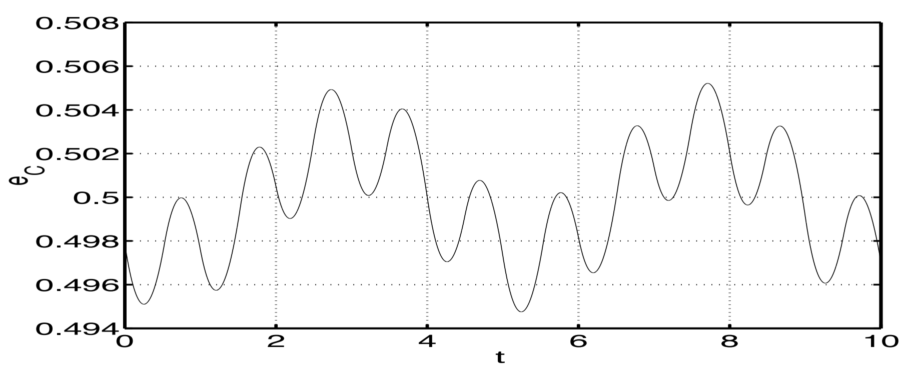

Figure 11: Unstable periodic solution of the real Buck converter with fixed modulation.

the theory can be observed clearly. Especially, note the superposition of two periodic signals with respectively the periods T, 2T, and T, 5T, which is in agreement with Table 3.

6 Conclusions

In this paper we have developed a general discrete-time theory of pulse-width-modulated switched power converters. We have investigated the existence, uniqueness and stability of stationary solutions. Also deviations from a stationary solution have been investigated. It has been shown that the so-called running and fixed PWM may introduce a very different behaviour of the feedback control system. Subharmonic oscillations are shown to be a special case of the normal unstable behaviour. The theory has been illustrated with the Buck converter.

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