Output Adaptive Model Reference Control of Linear Continuous State-Delay Plant

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Abstract

This paper develops a new approach for the output model reference adaptive control of linear continuous-time plants with state delays. The main idea is to include into the control law a feedforward component which compensates for the delayed states, in addition to output feedback. The feedforward is formed by special adaptively adjusted prefilters as a function of the delayed state of the reference model. The output feedback component is designed as for a plant without delay, but applied to the time-delay plant. Such a controller structure containing adaptive output error feedback and adaptive prefilters from the delayed reference model makes it possible to solve the problem of adaptive exact asymptotic output tracking under parametric uncertainties. The stability is analyzed using the Lyapunov-Krasovskii functional method. Simulation results show the effectiveness of the proposed scheme.

1 Introduction

The problem of output model reference control (MRAC) of continuous time plants with state delays is one of the potentially difficult problems of adaptive control theory. Taking into account the significance of time-delay systems, this problem is of fundamental importance from a theoretic point of view and also of great practical interest. To the authors' knowledge, there is no effective method available in the literature which is able to handle this problem. The difficulty in solving it emanates from the fact that the feedback connection of a plant with state delays produces a *transcendental transfer function.* For such systems it is impossible to apply the conceptually simple certainty equivalence approach in a straightforward way. Finding a controller structure that satisfies the *exact plant-model matching* or *perfect model following* conditions [1, 2] *remains a difficult open problem.* It is difficult to find a controller structure that admits perfect output tracking. In other words, this is so because the approach used in classical certainty equivalence adaptive control theory relies on the Kalman-Yakubovich lemma. To find a generalized version of the Kalman-Yakubovich lemma in the context of time-delay systems remains a difficult open problem, a problem that we do not intend to tackle. Our main idea for the design of output MRAC continuous time linear plants with state delays is to compensate the delayed states with delayed states of the reference model. For this purpose an *adaptive dynamic prefilter from the delayed reference model* will be introduced. Thus, the exact asymptotic output tracking problem will be solved, using output feedback, together with feedforward from the reference model.

We define the controller as the sum of two components, $u(t) = u_f(t) + u_g(t)$. The output feedback component u_f is designed as for a plant without delay, but applied to the time-delay plant, whereby the assumptions about the time-delay plant are such that $u_f(t)$ can be made stabilizing. The feedforward component u_g is the output of the adaptively adjusted prefilter.

2 Plant model and problem formulation

The class of plants we shall consider in this paper is of the form

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bu(t) + A_\tau x(t-\tau) \\ y(t) &= c^T x(t) \end{aligned} \tag{2.1}$$

where $x \in R^n$, $u(t) \in R$ and $y(t) \in R$. The constant matrices A, A_τ and vectors b, c of appropriate dimensions are unknown. $\tau \in R^+$ is the known time delay. We also assume that there exists vector $a_\tau^* \in R^n$ such that $A_\tau = ba_\tau^{*T}$.

We shall denote as $W_0(s)$ the transfer function of the system without delay. We assume that

$$W_0(s) = c^T(sI - A)^{-1}b = k_p \frac{N(s)}{D(s)} \tag{2.2}$$

satisfies the standard assumptions of adaptive control theory, e.g. [1, 3].

(A1) $D(s)$ is a monic polynomial of degree n.

(A2) $W_0(s)$ is minimum phase, i.e. $N(s)$ is Hurwitz.

(A3) The relative degree is one.

(A4) The sign of the high frequency gain k_p is known.

The minimum phase assumption (A2) is fundamental in MRAC schemes. Assumption (A3) focuses on the simplest case amenable to Lyapunov designs. The same idea can be extended to higher relative degree. For the case of relative degree greater than two, it is required to use "error augmentation" and/or "tuning error normalization", see e.g. [3].

Our objective is to determine a bounded control input $u(t)$ to the plant using a differentiator free controller so that the output $y(t)$ of the controlled plant (2.1) asymptotically exactly follows the output y_m of a stable reference model

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) + b_m r(t) \\ y_m(t) &= c_m^T x_m(t) \end{aligned} \tag{2.3}$$

where $x_m \in R^n$, $r(t)$ and $y_m(t) \in R$. $r(t)$ is the reference input which is assumed to be a uniformly bounded and piecewise continuous function of time. The transfer function of reference model is strictly positive real.

3 Proposed adaptive controller

We define the controller as the sum of two components

$$u(t) = u_f(t) + u_g(t) \tag{3.4}$$

where $u_f(t)$ is the feedback component and $u_g(t)$ is the feedforward component.

3.1 Feedback component

The differentiator free output feedback component $u_f(t)$ is designed as in conventional MRAC schemes for a plant without delay. This has been widely analyzed in the literature of adaptive control, see e.g. [1, 3]

$$u_f(t) = W_p(s)[u](t) + W_f(s)[y](t) + K_e e(t) + K_r r(t) \tag{3.5}$$

where

$$W_p(s) = K_p^T H(s),\ W_f(s) = K_f^T H(s)$$

$$H(s) = \frac{[1, s, \ldots, s^{n-2}]^T}{F(s)} \in R^{n-1}$$

$$K_p \in R^{n-1},\ K_f \in R^{n-1},\ K_e, K_r \in R$$

and $F(s)$ is any monic Hurwitz polynomial of degree $n-1$.

The state space realization of (3.5) is

$$\begin{aligned} \dot{x}_p(t) &= F x_p(t) + g u(t) \\ \dot{x}_f(t) &= F x_f(t) + g y(t) \\ K(t) &= [K_p^T, K_f^T, K_e, K_r]^T \\ \omega_f(t) &= [x_p^T(t), x_f^T(t), e(t), r(t)]^T \\ u_f(t) &= K^T(t)\omega_f(t) \end{aligned} \tag{3.6}$$

where

$$e(t) = y(t) - y_m(t) \tag{3.7}$$

is the tracking error, (F, g) is an asymptotically stable system in controllable canonical form with the elements in the last row equal to the coefficients of the characteristic polynomial of $W_0(s)$, $x_p(t) \in R^{n-1},\ x_f(t) \in R^{n-1},\ F \in R^{(n-1)\times(n-1)},\ g \in R^{n-1}$. The vector of adaptation gains K is the estimate of the unknown parameters $K_p^*,\ K_f^*,\ K_e^*, K_r^*$. The only difference is that in the regressor $\omega(t)$ the tracking error $e(t)$ is used instead of $y(t)$.

It is well known [1, 3] that a parameter vector $K^* = [K_p^{*T}, K_f^{*T}, K_e^*, K_r^*]^T$ exists such that if $K = K^*$, the transfer function of the plant without delays $W_0(s)$ together with the feedback controller matches that of the reference model $W_m(s)$ exactly.

3.2 Feedforward component

The reference model based feedforward component $u_g(t)$ is defined as the output of an adaptive dynamical prefilter. The prefilter in state space form and $u_g(t)$ are defined as follows

$$
\begin{aligned}
\dot{z}_m(t) &= F_d z_m(t) + g_d x_m(t-\tau) \\
u_g(t) &= -K_m^T(t)\omega_m(t) + K_z^T(t) z_m(t)
\end{aligned} \tag{3.8}
$$

where $\omega_m(t) = [y_m(t),\ x_m(t-\tau)^T]^T, \quad K_m(t) = [-K_{m1}(t),\ K_{m2}^T(t)]^T \in R^{n+1}$ and $K_z(t) \in R^{n(n-1)}$ are the time-varying adaptation gain vectors.

The state vector of the prefilter

$$
z_m(t) \in R^{n(n-1)} = [z_m^{1T}(t), \ldots, z_m^{nT}(t)]^T
$$

has the components

$$
\begin{aligned}
\dot{z}_m^1(t) &= F z_m^1(t) + g x_m^1(t-\tau), \\
&\vdots \\
\dot{z}_m^n(t) &= F z_m^n(t) + g x_m^n(t-\tau)
\end{aligned} \tag{3.9}
$$

where $F_d \in R^{n(n-1)\times n(n-1)} = \text{block-diag}(F)$, and $g_d \in R^{n(n-1)\times n} = \text{block-diag}(g)$. *This prefilter is the main contribution of our approach.*

4 Error equation

With the controller (3.5) and the parameter error $\Delta K(t) = K(t) - K^*$, the overall closed-loop system becomes

$$
\begin{aligned}
\dot{\hat{x}}(t) &= \hat{A}\hat{x}(t) + \hat{b}[\Delta K^T(t)\omega_f(t) + K_r^* r(t) - K_e^* y_m(t) + u_g(t)] + \bar{b} a_\tau^{*T} x(t-\tau) \\
y(t) &= \hat{c_m}^T \hat{x}
\end{aligned} \tag{4.10}
$$

where $\hat{x}(t) = [x^T(t), x_p^T(t), x_f^T(t)]^T \in R^{3n-2}$, $\hat{c}_m = [c^T, 0, 0]^T \in R^{3n-2}$ and

$$
\hat{A} = \begin{bmatrix} A + bK_y^* c^T & bK_p^{*T} & bK_f^{*T} \\ gK_e^* c^T & F + gK_p^{*T} & gK_f^{*T} \\ gc^T & 0 & F \end{bmatrix}, \quad \hat{b} = \begin{bmatrix} b \\ g \\ 0 \end{bmatrix}, \quad \bar{b} = \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix} \tag{4.11}
$$

As follows from Fig. 1 the term from the delay state $a_\tau^{*T} x(t-\tau)$ that enter at the input to the plant are not available as an input to the precompensator W_p from (3.5). Since

$$
\hat{W}_p(s) = \frac{1}{1 - W_p(s)} = \frac{N_m(s)}{N(s)} \tag{4.12}
$$

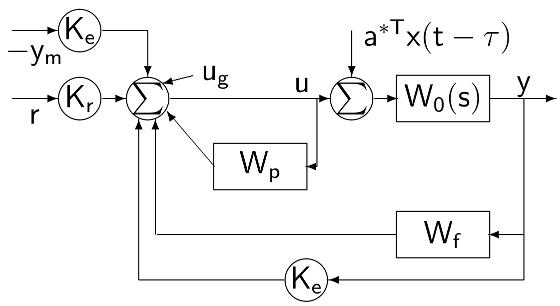

Figure 1: Block diagram of closed loop system

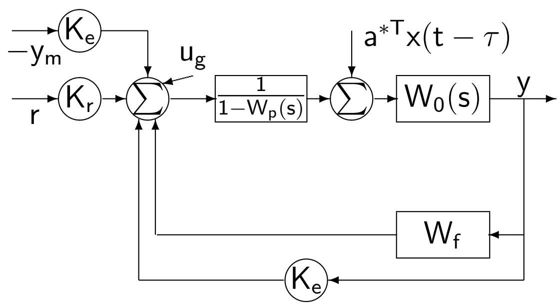

Figure 2: Block diagram of closed loop system - Equivalent form 1.

and since both $N_m(s)$ and $N(s)$ are assumed Hurwitz, we have that $\hat{W}_p(s)$ and $\hat{W}_p^{-1}(s)$ are stable transfer functions. Therefore we can reflect the signal $a_{\tau j}^{*T}x(t-\tau)$ to the input of the closed-loop system using standard transfer function manipulations as shown in Fig. 2 – 3.

In doing this, we introduce a new subsystem into the analysis, whose transfer function is $\hat{W}_p^{-1}(s) = 1 - W_p(s)$ with input $a_\tau^{*T}x(t-\tau)$, output $y_x(t)$

$$
\begin{aligned}
y_x &= (1 - W_p(s))a_\tau^{*T}x(t-\tau) \\
&= a_\tau^{*T}x(t-\tau) - W_p(s)[a_\tau^{*T}x(t-\tau)]
\end{aligned}
$$

Since a_τ^* is a constant, $y_x(t)$ can be rewritten as

$$
y_x(t) = a_\tau^{*T}x(t-\tau) - a_\tau^{*T}W_pI_n[x(t-\tau)] \tag{4.13}
$$

where I_n is $n \times n$ identity matrix. Consider the realization of (4.13)

$$
\begin{aligned}
\dot{z}_x(t) &= F_d z_x(t) + g_d x(t-\tau), \quad z_x(t_0) = 0 \\
y_x(t) &= a_\tau^{*T}x(t-\tau) - \hat{a}_\tau^{*T}z_x(t)
\end{aligned} \tag{4.14}
$$

where $z_x(t) \in R^{n(n-1)} = [z_x^{1T}(t), \ldots, z_x^{nT}(t)]^T$, $K_{pd} \in R^{n\times n(n-1)} =$ block-diag(K_p^{*T}), $\hat{a}_\tau^* = K_{pd}^T a_\tau^* \in R^{n(n-1)}$, and F_d, g_d from (3.8).

Defining $\hat{x}_m(t)$ as the reference model state corresponding to $\hat{x}(t)$ when the parameter errors are zero [1], the augmented error as $\hat{e}(t) = \hat{x}(t) - \hat{x}_m(t)$, we obtain the augmented

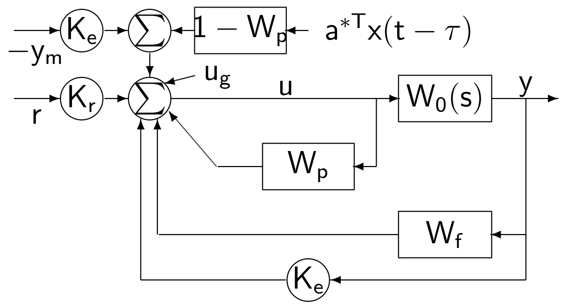

Figure 3: Block diagram of closed loop system - Equivalent form 2.

error model in the form

$$
\begin{aligned}
\dot{\hat{e}}(t) &= \hat{A}\hat{e}(t) + \hat{b}\left[\Delta K^T(t)\omega_f(t) - K_e^* y_m(t) + u_g(t) + a_\tau^{*T} L^T \hat{x}(t-\tau) - \hat{a}_\tau^{*T} z_x(t)\right] \\
\dot{z}_x(t) &= F_d z_x(t) + g_d L^T \hat{x}(t-\tau) \\
e(t) &= \hat{c}_m^T \hat{e}(t) \qquad (4.15)
\end{aligned}
$$

where $L = [I_{n\times n},\ 0_{n\times(n-1)},\ 0_{n\times(n-1)}]^T$.

Now we introduce $z_e(t)$ and $z_m(t)$ as $z_e(t) + z_m(t) = z_x(t)$. Then from (4.15), (3.8) we get

$$
\begin{aligned}
\dot{\hat{e}}(t) &= \hat{A}\hat{e}(t) + \hat{b} a_\tau^{*T} L^T \hat{e}(t-\tau) - \hat{b}\hat{a}_\tau^{*T} z_e(t) \\
&\quad + \hat{b}\Delta K^T(t)\omega_f(t) + \hat{b}\Delta K_m^T(t)\omega_m(t) + \hat{b}\Delta K_z^T(t) z_m(t) \\
\dot{z}_e(t) &= F_d z_e(t) + g_d L^T \hat{e}(t-\tau) \\
\dot{z}_m(t) &= F_d z_m(t) + g_d x_m(t-\tau) \\
e(t) &= \hat{c}_m^T \hat{e}(t) \qquad (4.16)
\end{aligned}
$$

where

$$
\begin{aligned}
\Delta K_m(t) &= [(K_{m1}(t) - K_e^*), (a_\tau^* - K_{m2}(t))^T]^T \\
\Delta K_z(t) &= K_z(t) - \hat{a}_\tau^* \\
\omega_m(t) &= [y_m(t),\ x_m(t-\tau)^T]^T. \qquad (4.17)
\end{aligned}
$$

5 Adaptation algorithms design

We denote the solutions of the system (4.16) by $\hat{e}(\Delta K(t), \Delta K_m(t), \Delta K_z(t))(t)$ and prove the following theorem

Theorem 5.1. *Consider the closed-loop system consisting of the plant described by (2.1), and the controller given by (3.4). Then all the signals in the system are bounded and the* *tracking error $e(t) \to 0$ as $t \to \infty$ if we choose the adaptive laws as*

$$\begin{aligned}\Delta\dot{K}(t) &= -\Gamma_1 e(t)\omega_f(t)\\ \Delta\dot{K}_m(t) &= -\Gamma_2 e(t)\omega_m(t)\\ \Delta\dot{K}_z(t) &= -\Gamma_3 e(t) z_m(t)\end{aligned} \tag{5.18}$$

where $\Gamma_1 = \Gamma_1^T > 0, \quad \Gamma_2 = \Gamma_2^T > 0, \quad \Gamma_3 = \Gamma_3^T > 0$ are constant matrices.

Proof: (Outline). We introduce the following Lyapunov-Krasovskii functional:

$$\begin{aligned}V(\hat{e}, z_e, \Delta K, \Delta K_m, \Delta K_z) &= \hat{e}^T(t)P_e\hat{e}(t) + z_e^T P_z z_e + \int_{t-\tau}^{t} \hat{e}^T(s)Q_z\hat{e}(s)ds\\ &\quad +(\Delta K - \hat{K}_1)^T\Gamma^{-1}(\Delta K - \hat{K}_1)\\ &\quad +\Delta K_m^T\Gamma^{-1}\Delta K_m + \Delta K_z^T\Gamma^{-1}\Delta K_z\end{aligned} \tag{5.19}$$

where $\hat{K}_1 = -r_0\hat{b}^T P_e$ and r_0 is a some constant.
The matrices P_e, P_z satisfy the equations

$$\begin{aligned}\hat{A}^T P_e + P_e\hat{A} &= -Q_e\\ P_e\hat{b} &= \hat{c}_m\\ F_d^T Q_e + Q_e F_d &= -Q_z\end{aligned} \tag{5.20}$$

where both Q_e and Q_z are positive definite matrices suitable dimensions. Compare the upper two equations of (5.20) with the Kalman-Yakubovich lemma. Note that the transfer function of the reference model (2.3) is strictly positive real and F_d is stable.
We shall compute the derivative of V along the solutions of the model transformation (4.16). We get

$$\dot{V} \le -q_e\hat{e}^T\hat{e} - q_z z_e^T z_e \le 0 \tag{5.21}$$

where q_e and q_z are some positive constants.
Furthermore, using standard arguments from stability theory [1, 3], we conclude that the solutions $\hat{e}$ are bounded and $e(t) \to 0$ as $t \to \infty$. A full proof is found in [4]

6 Example

We illustrate the application of the obtained adaptation algorithms for the following unstable second order plant

$$\begin{aligned}\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} &= \begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix}u(t)\\ &\quad + \begin{bmatrix}-0.2 & -0.1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1(t-4)\\ x_2(t-4)\end{bmatrix}\\ y(t) &= [1.0 \;\; 0.5]\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}\end{aligned} \tag{6.22}$$

The initial conditions of the plant states are $x(0) = [0 \;\; 0]^T$
It is required to design u such that the output $y(t)$ track the output $y_m(t)$ of the reference model

$$\begin{aligned} \dot{x}_m(t) &= \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} x_{mi}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r(t) \\ y_m(t) &= [1 \quad 1] x_m(t) \end{aligned} \tag{6.23}$$

According to (3.4), (3.5) and (3.8), the control law $u(t) = u_f + u_g$ is given by

Feedback component

$$\begin{aligned} \dot{x}_p(t) &= F x_p(t) + g u(t), \\ \dot{x}_f(t) &= F x_f(t) + g y(t), \\ u_f(t) &= K_p(t) x_p(t) + K_f(t) x_f(t) + K_e(t) e(t) + K_r(t) r(t) \\ &= K^T(t) \omega_f(t) \end{aligned} \tag{6.24}$$

where $K(t) = [K_p(t), K_f(t), K_e(t), K_r(t)]^T$, $\omega_f = [x_p(t), x_f(t), e(t), r(t)]^T$, $F = -1, g = 1$

Feedforward component

$$\begin{aligned} \begin{bmatrix} \dot{z}_{m1}(t) \\ \dot{z}_{m2}(t) \end{bmatrix} &= \begin{bmatrix} F & 0 \\ 0 & F \end{bmatrix} \begin{bmatrix} z_{m1}(t) \\ z_{m2}(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} g & 0 \\ 0 & g \end{bmatrix} \begin{bmatrix} x_{m1}(t-\tau) \\ x_{m2}(t-\tau) \end{bmatrix} \end{aligned} \tag{6.25}$$

$$u_g(t) = K_z^T(t) z_m(t) - K_m^T(t) x_m(t-\tau) + K_{ym} y_m(t). \tag{6.26}$$

Adaptation algorithms

$$\begin{aligned} \dot{K}(t) &= \gamma_k e(t) \omega_f \\ \dot{K}_m(t) &= \gamma_m e(t) x_m(t-\tau) \\ \dot{K}_{ym}(t) &= \gamma_m e(t) y_m(t) \\ \dot{K}_z(t) &= \gamma_z e(t) z_m(t). \end{aligned} \tag{6.27}$$

where $\gamma_k = \gamma_m = \gamma_z = 9$. Simulation results are shown in Figures 4 – 6. where we show the time responses of the tracking error e, the control u, the output of plant y, the output of the reference model y_m, and the adaptive control parameters K and K_m, K_z for a square wave reference command.

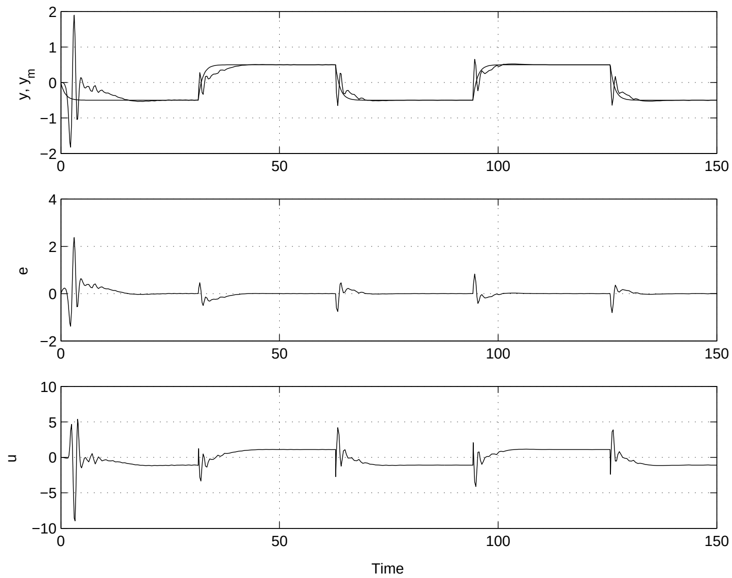

Figure 4: The time history of the plant and reference model outputs and the error and control signals, respectively.

7 Conclusion

In this paper, by using adaptively adjusted prefilters from the state reference model, we have presented a new approach to the design of output adaptive controllers for a class of uncertain time-delay systems to track dynamic inputs generated from a reference model. The main idea is the introduction in the control law of a feedforward component which compensates the delayed states, in addition to output feedback. The feedforward is formed by special adaptively adjusted prefilters as a function of the delayed state of the reference model. Such a controller structure containing adaptive output feedback and adaptive prefilters from the delayed reference model makes it possible to solve the problem of *adaptive exact asymptotic output tracking* under parametric uncertainties. A simple example is given to show the potential of the proposed techniques. The extension of a proposed approach to the design of model reference adaptive output feedback controller for systems of arbitrary relative degree is a current research topic.

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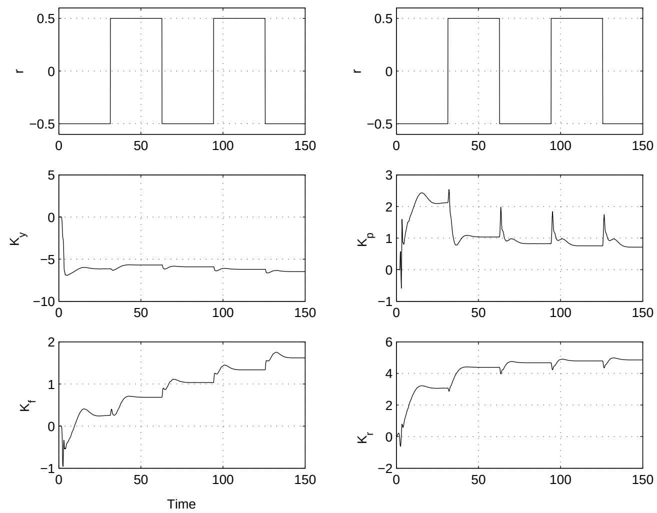

Figure 5: The graphs show the command signal and adjustment parameters time history.

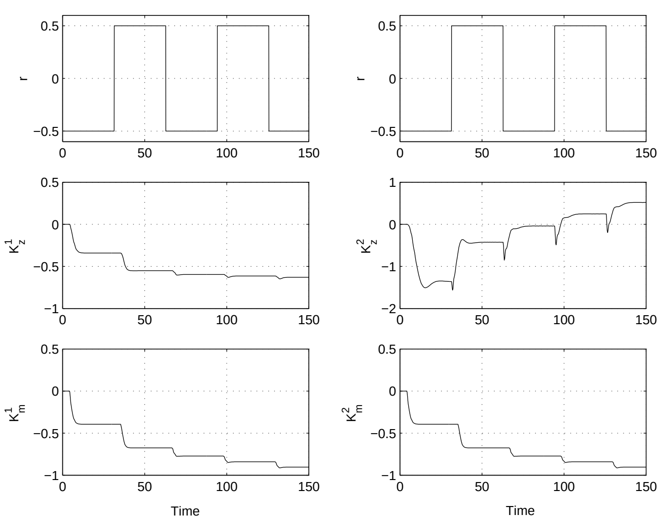

Figure 6: The graphs show the command signal and adjustment parameters time history.